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AN INVESTIGATION OF GAIN-TUNED ACTIVE FILTERS

A THESIS

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AN INVESTIGATION OF GAIN-TUNED ACTIVE FILTERS

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Date approved by Chairman: 11-21-72

DEDICATION

I would like to dedicate this dissertation
to my wife, Porntip Bruminhent.

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF ILLUSTRATIONS

SUMMARY

The purpose of this investigation is to study a class of tunable active filters. The active filters of interest are realized by using ideal VCVS as active elements. The center-frequency or the cut-off frequency of the filter is to be tuned within a range by varying the controlled gains of the VCVS. Since active filters can be tuned by varying their active elements, they can be used to advantage in the design of electronically tuned filters.

Tunable active filters have found applications in such areas as the analysis of seismic data, pattern recognition, and instrumentation. As the technology of tunable active filters improves, they should find additional applications in many other areas such as control, data transmission, bioelectronics, and communications. Tunable active filters have received a great deal of attention and many investigations have been made in this area.[8, 10, 11, 12]

The first class of tunable filter, the constant bandwidth, can be easily realized by using only one VCVS. It is possible to tune the center-frequency or the cut-off frequency of the filter by varying the gain of the VCVS. Thus, the realization of active tunable constant-bandwidth filter is quite simple.

The other class of tunable filter, the constant percentage bandwidth, has been found particularly useful in the analysis of sound extending over a broad frequency range.[13, 14, 15] This is the class of filter to be investigated in this research.

The synthesis of the network is confined to that of second-order transfer functions as needed for low-pass, high-pass, or band-pass functions. This is to alleviate the high sensitivity of the pole positions to changes in the elements and to arrive at simple circuits.

The approach is to find circuits containing controlled gains K_0's such that the coefficients of s^2, s^1, and s^0 terms in the characteristic equation can be dominated by the terms K_0^2, K_0^1, and K_0^0 or in reversing order K_0^0, K_0^1, and K_0^2. For such circuits, it is possible to tune the center-frequency or the cut-off frequency and at the same time keep the percentage bandwidth (or equivalently, Q) constant by simply changing the value of K_0. It was found that:

1. At least two VCVS and a five-node network are needed.

2. There is no practical advantage by increasing either the number of VCVS or the number of nodes.

3. The controlled gains, K_0's, of the VCVS should be kept at their minimum values so that the VCVS can be easily realized.

4. The ideal characteristics of the filters are approached when the controlled gains of the VCVS are very much larger than unity. However, finite values of K_0's have to be used in actual circuits and result in slight changes in Q when the center-frequency or the cut-off frequency is tuned.

5. The minimum required gains are found so that the change in Q will be within a given tolerance.

6. The minimum required gains can be achieved with two VCVS and a five-node network.

7. The minimum required gains do not change when the number of nodes and the number of VCVS are increased.

8. Two VCVS and five-node network are used in realizing the filters.

Using the approach described, eighty circuits are found for the band-pass sections, and thirty-three circuits each are found for the low-pass and the high-pass sections.

Minimization on the magnitudes of the controlled gains is made to the derived sections. Further minimization on the number of the passive elements is done and results in fourteen final circuits for the band-pass sections, and six circuits each for the low-pass and the high-pass sections. The input and output impedances of these final circuits are derived so that they can be utilized to the greatest advantage.

Numerical examples are included to illustrate the designing of the band-pass, low-pass, and high-pass sections. An actual circuit was built in the laboratory using two commercially available operational amplifiers. The results of the experiment are presented.

CHAPTER I

INTRODUCTION

In the past two decades, active network theory has received considerable attention. Since the early work of Linvill,[1] and Sallen and Key,[2] substantial progress has been made in the area of active filters.

Active networks can not only realize all network functions realizable by passive RLC networks, but also functions that are not realizable by the passive RLC networks. In addition, the synthesis of the active network is simpler than the synthesis of the passive network. The size of an active filter is usually smaller than the size of the passive filter of the same specification at low frequencies. With the advent of integrated-circuit technology, active filters frequently cost less than the corresponding passive filters.

Many active filter realizations have been suggested[3,4,5,6,7] based on the use of some basic active elements such as the negative impedance converter (NIC), the gyrator, controlled sources, the operational amplifier, and the automatic phase-locked loop. The choice of using any one of these active elements depends upon the application. The question of which active element is most desirable for what application appears to be a matter of continuing development in the years to come.

In recent years, tunable filters have been playing increasing roles in signal processing. In realizing a tunable filter, active or passive elements must be varied in order to vary the frequency response of the filter. Resistors and capacitors can be varied mechanically, or by using specialized devices such as raysistors and varicap diodes. These methods present practical problems, especially where high accuracy is important. Since active filters can be tuned by varying their active elements, they can be used to advantage in the design of electronically tuned filters.

Tunable active filters have found applications in such areas as the analysis of seismic data, pattern recognition, and instrumentations. As the technology of tunable active filters improves, they should find additional applications in many other areas such as control, data transmission, bioelectronics, and communications.

Some investigations had been made in the area of the tunable active filter. Kerwin and Shaffer[8] demonstrated a circuit which is tunable both in selectivity and in frequency. Their circuit uses the state-variable approach of Kerwin, Huelsman, and Newcomb.[9] The filter is composed of four operational amplifiers. Three sets of resistors are used to independently tune the center frequency, Q, and the gain at the center frequency. One numerical example is shown in which the frequency range varies from 4421 to 39789 Hz with slight changes in measured Q and the gain at the center frequency. However, a close look at the approach of Kerwin and Shaffer will show that the circuit is almost totally changed when the center frequency is tuned from one value to another. The method is equivalent to designing the whole new circuit for each frequency.

Bruton[10] used the general impedance converter (GIC) as the basic active elements for the tunable low-pass active filter. He used four-quadrant analogue multipliers as the nonlinear network elements. The concept of scale impedances and square-scale impedances is used; that is, the inductors, capacitors, and resistors are scaled into resistors, super-capacitors,[+] and capacitors, respectively. The cut-off frequency of the filter is varied by means of a tuning voltage. A numerical example for a low-pass filter with the pass-band ripple of 1 db is shown. The required tuning voltage is from -2.5 to +2.5 volts for tuning the cut-off frequency from 270 to 450 Hz. If varicap diodes had been used, we would have expected the similar result by using any conventional method of realizing active filters. This is because when the control voltage is changed, the capacitance is also changed and so is the cut-off frequency.

Kaehler[11] has proposed a method of electronically tuning a filter transfer function by periodically switching network elements. The coefficients of the transfer function become functions of the pulsewidth-to-period ratios of the switching elements. The cut-off frequency of the filter is tuned by varying the pulsewidth-to-period ratio.

Bruton[12] applied the RC networks containing periodically-switched conductances with the frequency-scaling network functions. His method is based on the evaluation of the response of a capacitance voltage to the application of an impulse at the input part of the network. By using periodically-switched conductances, the impulse response can be amplitude and time scaled; and, therefore, the transfer function can be frequency scaled.

[+]The impedance of the super-capacitor D is $1/(s^2D)$.

Recent studies of active networks have shown that the poles of the transfer function of an active network can be very sensitive to the parameters of its active elements. This fact strongly suggests that the gains of the active elements can be used to tune the active filter. It is the purpose of this research to investigate this facet of active network theory.

The tunable band-pass filters fall into two broad classes: (1) The constant-bandwidth filters in which the difference between the upper cut-off frequency and the lower-cutoff frequency remains a constant over the tuning range. (2) The constant-percentage bandwidth (constant Q*) filters in which the ratio between upper cut-off frequency to the lower cut-off frequency remains a constant over the tuning range. The latter type of filter has been found particularly useful in the analysis of sounds extending over a broad frequency range[13,14,15].

If the first type of filter, constant bandwidth, is needed, the gain-tuned active filter can be realized by using only one controlled source. Hence, this problem is a relatively simple one. One circuit for that purpose is shown in Figure 1. The voltage transfer function of this circuit is

---

*For a second-order transfer function of the form

$$\frac{V_{out}}{V_{in}} = \frac{A(s)}{as^2+bs+c}$$

Q is defined by

$$Q = \frac{\sqrt{ac}}{b}$$

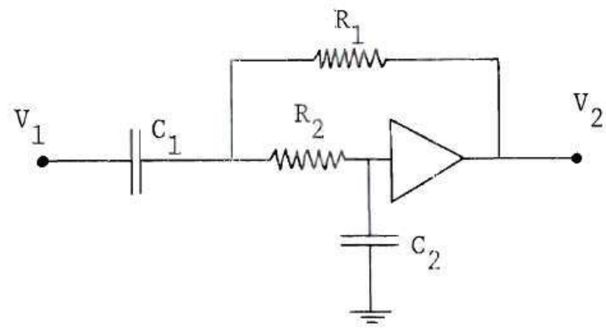

Figure 1. A Tunable Band-pass Filter

$$\frac{V_2}{V_1} = \frac{-G_2C_1Ks}{C_1C_2s^2+s(G_1C_2+G_2C_2+G_2C_1)+G_1G_2(1-K)} \tag{1}$$

$$\omega = \text{center frequency} \tag{2}$$

$$= \sqrt{\frac{G_1G_2(1-K)}{C_1C_2}}$$

$$Q = \frac{\sqrt{G_1G_2C_1C_2(1-K)}}{G_1C_2+G_2C_2+G_2C_1} \tag{3}$$

$$\text{Bandwidth} = \frac{G_1C_2+G_2C_2+G_2C_1}{C_1C_2} \tag{4}$$

Equation (4) indicates that the bandwidth of this filter is independent of the amplifier gain, K.

The purpose of this research is to study a class of constant-Q tunable active filters. The active filters of interest are realized by using voltage-controlled voltage sources (VCVS). The center frequency or the cut-off frequency of the filter is to be tuned within a range by varying the gains of the VCVS.

CHAPTER II

GAIN-TUNED ACTIVE FILTERS
USING VOLTAGE-CONTROLLED VOLTAGE SOURCES

In the area of active filter synthesis, it is sufficient and desirable to confine the study to the realization of various second-order transfer functions needed for low-pass, high-pass and band-pass functions in normalized form. This is to alleviate three fundamental problems of active RC synthesis:

(1) Attempts to obtain higher than second-order functions in a single structure with one or more active elements lead to a higher sensitivity of the pole positions to changes in the elements than necessary.

(2) The pole positions become highly sensitive functions of polynomials, thereby making it extremely difficult to realize a desired set of poles by achieving a given set of polynomial coefficients. This problem is particularly serious in the use of state-variable synthesis methods.

(3) Higher-order circuits are difficult to design. When higher-order systems are needed, they can be synthesized by the cascade connection of the first- and second-order functions. (Of course, any first-order functions with real poles can be conveniently realized by standard methods of passive RC network synthesis.)

In this research, we will concern ourselves primarily with active RC networks in which the active elements are VCVS. The VCVS will be assumed to have ideal characteristics, that is, infinite input impedance, zero output impedance, zero reverse transmission and ideal phase shift (either zero or 180°). The passive elements will be resistors and capacitors only. We will consider only the lumped, linear and time-invarying resistors and capacitors, for these are the prime passive elements of linear integrated circuits.

One possible solution of finding gain-tuned active filters with Q being kept constant while using the gains of the VCVS to vary the center frequency is to find a circuit such that the voltage-transfer function is of the form:

$$\frac{V_o}{V_i} = \frac{ds}{a^2K_0^2s^2 + bK_0s + 1} \tag{5}$$

We then have,

$$\omega_0 = \frac{1}{aK_0} \tag{6}$$

$$Q_0 = \frac{a}{b} \tag{7}$$

$$A_0 = \text{gain at } \omega_0 \tag{8}$$

$$= \frac{d}{bK_0}$$

If we could obtain the circuit that gives the voltage-transfer function as in (5), we can use K_0 to control ω_0 while keeping Q_0 constant.

An alternative voltage-transfer function will be

$$\frac{V_o}{V_i} = \frac{d's}{a'^2s^2 + b'K_0s + K_0^2} \tag{9}$$

The corresponding equations for ω_0, Q_0, and A_0 are:

$$\omega_0 = \frac{K_0}{a'} \tag{10}$$

$$Q_0 = \frac{a'}{b'} \tag{11}$$

$$A_0 = \frac{d'}{b'K_0} \tag{12}$$

Equations (10), (11), and (12) are similar to (6), (7), and (8) except that in the later set ω_0 is directly, instead of inversely, proportional to K_0.

For practical reasons, the gains of the VCVS should be made as small as possible. One of the advantages for using low gains is that the VCVS can be realized more easily. Low gains also reduce the likelihood of oscillations due to parasitic feedbacks. On the other hand, since the approximation that these gains are much greater than unity will be used, this approximation would be improved if high gains are used. Hence, a compromise must be made between these two conflicting requirements. This compromise is accomplished by making the gains just large enough to satisfy the approximation requirement.

It will be shown in the following development that:

(I) The minimum required number of VCVS is two.

(II) The minimum required number of nodes in each filter section is five.

(III) The minimum required gain* for the VCVS does not change when the number of nodes is increased.

Proof of Statement (I)

The denominators of (5) and (9) have both the terms K_0 and K_0^2. We cannot use only one VCVS with gain K_0 to give both the terms K_0 and K_0^2. Therefore the minimum required number of VCVS is two.

Proof of Statement (II)

With two VCVS and a flexible number of nodes, the network can be realized in the configurations shown in Figures 2, 3, and 4. We will consider first Figure 2. The output, V_{out}, can be taken from any node n, $n \geq 3$. We shall use the following notation:

y_{rr} = the sum of all generalized admittances connected to node r.

y_{pq} = the negative of the sum of all generalized admittances connected between nodes p and q.

V_r = the Laplace transform of the voltage at node r with respect to ground (node n+1).

Summing the currents at nodes 1 to n gives the following equation:

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*This is the minimum value of K_0, as defined in (5) or (9), which is required for a certain tolerance of the changes in Q_0 when the filter is tuned.

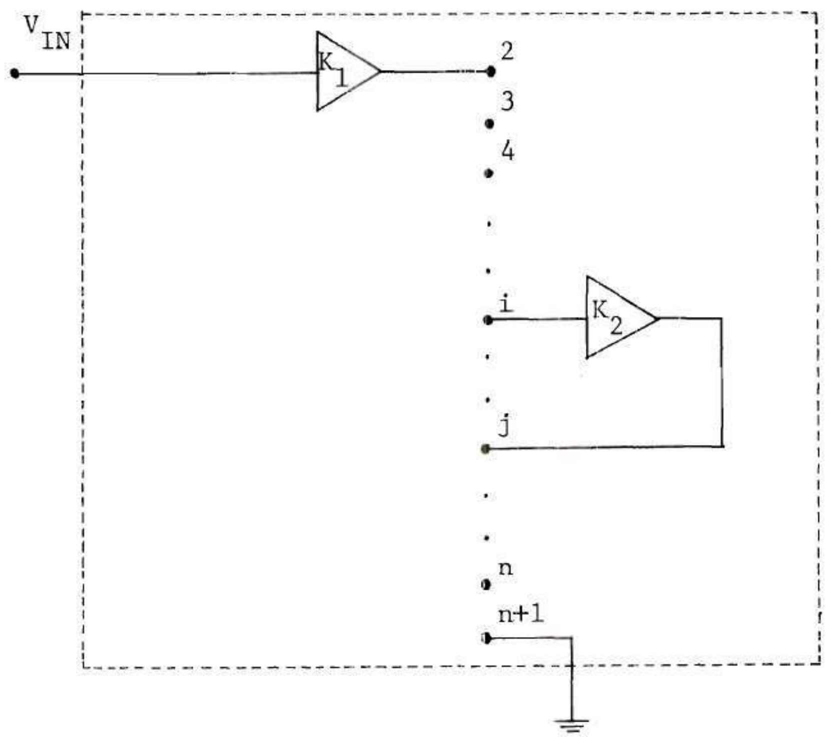


Figure 2. The Configuration #1 of the Two-VCVS Network

$$\left[\, 0 \,\right] = \begin{bmatrix} y_{11} & y_{12} & y_{13} & \cdot & \cdot & \cdot & y_{1i} & \cdot & \cdot & y_{1j} & \cdot & \cdot & y_{1n} \\ y_{21} & y_{22} & y_{23} & \cdot & \cdot & \cdot & y_{2i} & \cdot & \cdot & y_{2j} & \cdot & \cdot & y_{2n} \\ y_{31} & y_{32} & y_{33} & \cdot & \cdot & \cdot & y_{3i} & \cdot & \cdot & y_{3j} & \cdot & \cdot & y_{3n} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ y_{i1} & y_{i2} & y_{i3} & \cdot & \cdot & \cdot & y_{ii} & \cdot & \cdot & y_{ij} & \cdot & \cdot & y_{in} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ y_{j1} & y_{j2} & y_{j3} & \cdot & \cdot & \cdot & y_{ji} & \cdot & \cdot & y_{jj} & \cdot & \cdot & y_{jn} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ y_{n1} & y_{n2} & y_{n3} & \cdot & \cdot & \cdot & y_{ni} & \cdot & \cdot & y_{nj} & \cdot & \cdot & y_{nn} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ \cdot \\ \cdot \\ V_i \\ \cdot \\ \cdot \\ V_j \\ \cdot \\ \cdot \\ V_n \end{bmatrix} \tag{13}$$

We also have

$$V_2 = K_1V_1 = K_1V_{in} \tag{14}$$

and

$$V_j = K_2V_i \tag{15}$$

Therefore (13) can be rewritten as

$$[0] = \begin{bmatrix} y_{11}+K_1y_{12} \\ y_{31}+K_1y_{32} \\ \cdot \quad \cdot \quad \cdot \\ y_{i1}+K_1y_{i2} \\ \cdot \quad \cdot \quad \cdot \\ y_{n1}+K_1y_{n2} \end{bmatrix} V_{IN} + \begin{bmatrix} y_{13} & \cdot & \cdot & (y_{1i}+K_2y_{1j}) & \cdot & \cdot & y_{1n} \\ y_{33} & \cdot & \cdot & (y_{3i}+K_2y_{3j}) & \cdot & \cdot & y_{3n} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ y_{i3} & \cdot & \cdot & (y_{ii}+K_2y_{ij}) & \cdot & \cdot & y_{in} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ y_{n3} & \cdot & \cdot & (y_{ni}+K_2y_{nj}) & \cdot & \cdot & y_{nn} \end{bmatrix} \begin{bmatrix} V_3 \\ V_4 \\ \cdot \\ \cdot \\ V_i \\ \cdot \\ \cdot \\ V_n \end{bmatrix} \tag{16}$$

When an output is taken from any node n, n $\geq$ 3, the denominator of the voltage-transfer function is

$$D(s) = \begin{vmatrix} y_{13} & y_{14} & \cdot & \cdot & (y_{1i}+K_2y_{1j}) & \cdot & \cdot & y_{1n} \\ y_{33} & y_{34} & \cdot & \cdot & (y_{3i}+K_2y_{3}j) & \cdot & \cdot & y_{3n} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ y_{i3} & y_{i4} & \cdot & \cdot & (y_{ii}+K_2y_{ij}) & \cdot & \cdot & y_{in} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ y_{n3} & y_{n4} & \cdot & \cdot & (y_{ni}+K_2y_{nj}) & \cdot & \cdot & y_{nn} \end{vmatrix} \tag{17}$$

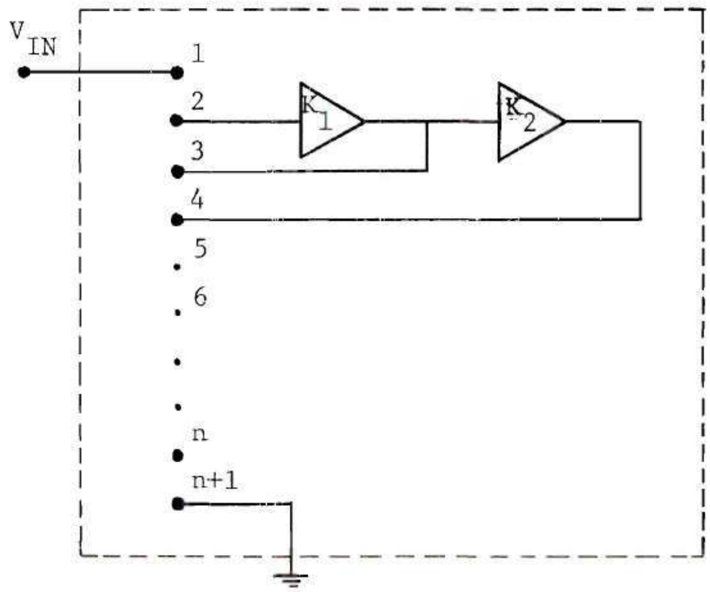


Figure 3. The Configuration #2 of the Two-VCVS Network

From (17), we see that D(s) is composed only of the term $(K_2)^0$ and $(K_2)^1$. We cannot have both the terms K_0 and K_0^2 simultaneously. Therefore, if one of the VCVS is connected to the input voltage, we cannot have D(s) of (5) or (9).

We will consider next the network of Figure 3.

Putting $$V_3 = K_1 V_2 \tag{18}$$

and $$V_4 = K_2 V_3 = K_1 K_2 V_2 \tag{19}$$

Summing the currents at nodes 2 to n gives the following equation:

$$[0] = \begin{bmatrix} y_{21} \\ y_{51} \\ y_{61} \\ \cdot \\ \cdot \\ y_{n1} \end{bmatrix} V_1 + \begin{bmatrix} (y_{22}+K_1y_{23}+K_1K_2y_{24}) & y_{25} & y_{26} & \cdot & \cdot & y_{2n} \\ (y_{52}+K_1y_{53}+K_1K_2y_{54}) & y_{55} & y_{56} & \cdot & \cdot & y_{5n} \\ (y_{62}+K_1y_{63}+K_1K_2y_{64}) & y_{65} & y_{66} & \cdot & \cdot & y_{6n} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ (y_{n2}+K_1y_{n3}+K_1K_2y_{n4}) & y_{n5} & y_{n6} & & & y_{nn} \end{bmatrix} \begin{bmatrix} V_2 \\ V_5 \\ V_6 \\ \cdot \\ \cdot \\ \cdot \\ V_n \end{bmatrix} \tag{20}$$

When an output is taken from any node n, $n \geq 2$, the denominator of the voltage-transfer function is

$$D(s) = \begin{vmatrix} (y_{22}+K_1y_{23}+K_1K_2y_{24}) & y_{25} & y_{26} & \cdot & \cdot & y_{2n} \\ (y_{52}+K_1y_{53}+K_1K_2y_{54}) & y_{55} & y_{56} & \cdot & \cdot & y_{5n} \\ (y_{62}+K_1y_{63}+K_1K_2y_{64}) & y_{65} & y_{66} & \cdot & \cdot & y_{6n} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ (y_{n2}+K_1y_{n3}+K_1K_2y_{n4}) & y_{n5} & y_{n6} & & & y_{nn} \end{vmatrix} \tag{21}$$

If n = 4,

$$D(s) = y_{22}+K_1y_{23}+K_1K_2y_{24} \tag{22}$$

From (22), even though we can have the terms K_0 and K_0^2 simultaneously by letting $K_1 = -K_0$ and $K_2 = K_0$, we fail to obtain the term with s^2. Therefore the configuration of Figure 3 cannot yield a D(s) of the second order.

If $n = 5$,

$$D(s) = y_{22}y_{55}-y_{25}y_{52}+K_1(y_{23}y_{55}-y_{25}y_{53})+K_1K_2(y_{24}y_{55}-y_{25}y_{54}) \quad (23)$$

From (23) we found that it is possible to have D(s) in quadratic form. And with proper values of K_1 and K_2, we can have D(s) in the forms of (5) or (9).

There are three possible choices of K's:

(i)
$$K_1 = -K_0 \quad (24)$$

$$K_2 = K_0 \quad (25)$$

(ii)
$$K_1 = -K_0^2 \quad (26)$$

$$K_2 = \frac{1}{K_0} \quad (27)$$

(iii) If we let

$$V_4 = K_2V_2 \quad (28)$$

Then another choice of K's is

$$K_1 = -K_0 \quad (29)$$

$$K_2 = -K_0^2 \quad (30)$$

The synthesis for three choices of K's yield the same network with some modifications.

We will analyze the circuit with choice (i) of K's as given in (24) and (25). From (23)

$$D(s) = (y_{22}y_{55}-y_{25}y_{52})-K_0(y_{23}y_{55}-y_{25}y_{53})-K_0^2(y_{24}y_{55}-y_{25}y_{54}) \tag{31}$$

$$= D_1(s) + K_0D_2(s) + K_0^2D_3(s) \tag{32}$$

where

$$D_1(s) = y_{22}y_{55}-y_{25}y_{52} \tag{33}$$

$$D_2(s) = y_{25}y_{53}-y_{23}y_{55} \tag{34}$$

$$D_3(s) = y_{25}y_{54}-y_{24}y_{55} \tag{35}$$

We will define:

$$\sum_{D_r(r = 1,2,3)} C^2 = \text{the coefficient of } s^2\text{-term in } D_r(s) \tag{36}$$

$$\sum_{D_r(r = 1,2,3)} CG = \text{the coefficient of } s\text{-term in } D_r(s) \tag{37}$$

$$\sum_{D_r(r = 1,2,3)} G^2 = \text{the coefficient of } s^0\text{-term in } D_r(s) \tag{38}$$

If the voltage-transfer function of (5) is desired, $D_3(s)$ must be a function of s^2 only. We have

$$D_1(s) = s^2\sum_{D_1} C^2 + s\sum_{D_1} CG + \sum_{D_1} G^2 \tag{39}$$

$$D_2(s) = s^2\sum_{D_2} C^2 + s\sum_{D_2} CG \tag{40}$$

$$D_3(s) = s^2\sum_{D_3} C^2 \tag{41}$$

Thus,

$$D(s) = s^2(\sum_{D_1} C^2 + K_0\sum_{D_2} C^2 + K_0^2\sum_{D_3} C^2) + s(\sum_{D_1} CG + K_0\sum_{D_2} CG) + \sum_{D_1} G^2 \tag{42}$$

We use the assumption that $K_0 >> 1$ and approximate the coefficient of s^2-term by the term with K_0^2 only.

$$D(s) = s^2K_0^2\sum_{D_3} C^2 + s(\sum_{D_1} CG + K_0\sum_{D_2} CG) + \sum_{D_1} G^2 \tag{43}$$

We let

$$\sum_{D_1} CG = AK_0\sum_{D_2} CG \quad , \quad A > 0 \tag{44}$$

Then

$$D(s) \quad = \quad s^2K_0^2\sum_{D_3} C^2 + s(1+A)K_0\sum_{D_2} CG + \sum_{D_1} G^2 \tag{45}$$

We have

$$Q_0 \quad = \quad \frac{\sqrt{\sum_{D_3} C^2 \sum_{D_1} G^2}}{(1+A)\sum_{D_2} CG} \tag{46}$$

From (44) and (46) we see that if K_0 is very large, then $A \simeq 0$ and Q_0 is independent of K_0.

We substitute (44) into (46),

$$Q_0 \quad = \quad \frac{\sqrt{\sum_{D_3} C^2 \sum_{D_1} G^2}}{\sum_{D_1} CG} \cdot \frac{AK_0}{(1+A)} \tag{47}$$

or

$$K_0 \quad = \quad Q_0\frac{(1+A)}{A} \cdot \frac{\sum_{D_1} CG}{\sqrt{\sum_{D_3} C^2 \sum_{D_1} G^2}} \tag{48}$$

We will show that

$$D_3(s) \subset D_1(s) \tag{49}$$

From (33) and (35)

$$D_1(s) = y_{22}y_{55} - y_{25}y_{52} \tag{33}$$

$$D_3(s) = y_{25}y_{54} - y_{24}y_{55} \tag{35}$$

In order for the filter to be stable, both $D_1(s)$ and $D_3(s)$ must be positive definite.

$$D_3(s) = y_{25}y_{54}-y_{24}y_{55} \leq y_{22}y_{55}-y_{25}y_{52} = D_1(s) \tag{50}$$

That is

$$D_3(s) \subset D_1(s) \tag{51}$$

From (51) it is also true that

$$\sum_{D_3} C^2 \subset \sum_{D_1} C^2 \tag{52}$$

For $D_1(s)$ as in (33), it is easy to show that we always have

$$\min\left(\frac{\sqrt{\sum_{D_1} C^2 \sum_{D_1} G^2}}{\sum_{D_1} CG}\right) = \frac{1}{2} \tag{53}$$

From (48), (52), and (53)

$$K_0 = Q_0 \frac{(1+A)}{A} \cdot \frac{\sum\limits_{D_1} CG}{\sqrt{\sum\limits_{D_3} C^2 \sum\limits_{D_1} G^2}} \tag{54}$$

$$\geq Q_0 \frac{(1+A)}{A} \cdot \frac{\sum\limits_{D_1} CG}{\sqrt{\sum\limits_{D_1} C^2 \sum\limits_{D_1} G^2}}$$

$$\geq 2Q_0 \frac{(1+A)}{A}$$

$$K_{0_{min}} = 2Q_0 \frac{(1+A)}{A} \tag{55}$$

If the voltage-transfer function of form (9) is desired, we must let $D_3(s)$ be a function of s^0 only. That is,

$$D_1(s) = \sum_{D_1} G^2 + s\sum_{D_1} GC + s^2\sum_{D_1} C^2 \tag{56}$$

$$D_2(s) = \sum_{D_2} G^2 + s\sum_{D_2} CG \tag{57}$$

$$D_3(s) = \sum_{D_3} G^2 \tag{58}$$

$$D(s) = s^2\sum_{D_1} C^2 + s(\sum_{D_1} CG + K_0\sum_{D_2} CG) \tag{59}$$

$$+ \sum_{D_1} G^2 + K_0\sum_{D_2} G^2 + K_0^2\sum_{D_3} G^2$$

We use the assumption that $K_0 >> 1$ and approximate the coefficient of s^0-term by the term with K_0^2 only.

$$D(s) = s^2\sum_{D_1} C^2 + s(\sum_{D_1} CG + K_0\sum_{D_2} CG) + K_0^2\sum_{D_3} G^2 \tag{60}$$

From (44),

$$D(s) = s^2\sum_{D_1} C^2 + s(1+A)K_0\sum_{D_2} CG + K_0^2\sum_{D_3} G^2 \tag{61}$$

We have

$$Q_0 = \frac{\sqrt{\sum_{D_1} C^2 \sum_{D_3} G^2}}{(1+A)\sum_{D_2} CG} \tag{62}$$

Equation (62) is similar to (46), we substitute (44) into (62) and rearrange the equation to be in the form of (48).

$$K_0 = Q_0 \frac{(1+A)}{A} \cdot \frac{\sum_{D_1} CG}{\sqrt{\sum_{D_1} C^2 \sum_{D_3} G^2}} \tag{63}$$

From (51) we have shown that $D_3(s) \subset D_1(s)$, therefore,

$$\sum_{D_3} G^2 \subset \sum_{D_1} G^2 \tag{64}$$

$$K_0 = Q_0 \frac{(1+A)}{A} \cdot \frac{\sum\limits_{D_1} CG}{\sqrt{\sum\limits_{D_1} C^2 \sum\limits_{D_3} G^2}} \tag{65}$$

$$\geq Q_0 \frac{(1+A)}{A} \cdot \frac{\sum\limits_{D_1} CG}{\sqrt{\sum\limits_{D_1} C^2 \sum\limits_{D_1} G^2}}$$

$$\geq 2Q \frac{(1+A)}{A}$$

$$K_{0_{min}} = 2Q_0 \frac{(1+A)}{A} \tag{66}$$

We see that the condition for $K_{0_{min}}$ remains the same for the voltage-transfer functions of (5) and (9).

In general, for any n, if the choices of K's are chosen as in (24) and (25)

$$D_1(s) = \begin{vmatrix} y_{22} & y_{25} & y_{26} & \cdot & \cdot & y_{2n} \\ y_{52} & y_{55} & y_{56} & \cdot & \cdot & y_{5n} \\ y_{62} & y_{65} & y_{66} & \cdot & \cdot & y_{6n} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ y_{n2} & y_{n5} & y_{n6} & \cdot & \cdot & y_{nn} \end{vmatrix} \quad (67)$$

$$D_2(s) = - \begin{vmatrix} y_{23} & y_{25} & y_{26} & \cdot & \cdot & y_{2n} \\ y_{53} & y_{55} & y_{56} & \cdot & \cdot & y_{5n} \\ y_{63} & y_{65} & y_{66} & \cdot & \cdot & y_{6n} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ y_{n3} & y_{n5} & y_{n6} & \cdot & \cdot & y_{nn} \end{vmatrix} \quad (68)$$

$$D_3(s) = - \begin{vmatrix} y_{24} & y_{25} & y_{26} & \cdot & \cdot & y_{2n} \\ y_{54} & y_{55} & y_{56} & \cdot & \cdot & y_{5n} \\ y_{64} & y_{65} & y_{66} & \cdot & \cdot & y_{6n} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ y_{n4} & y_{n5} & y_{n6} & \cdot & \cdot & y_{nn} \end{vmatrix} \quad (69)$$

For either the voltage-transfer functions of (5) or (9), the proof will follow that for n = 5. In both forms, we need to show that

$$D_3(s) < D_1(s) \quad (70)$$

But from (67), we see that $D_1(s)$ contains the term $(y_{22}y_{55}y_{66}\cdots y_{nn})$ which includes all terms in $D_2(s)$ and $D_3(s)$. Therefore we always have (70) satisfied.

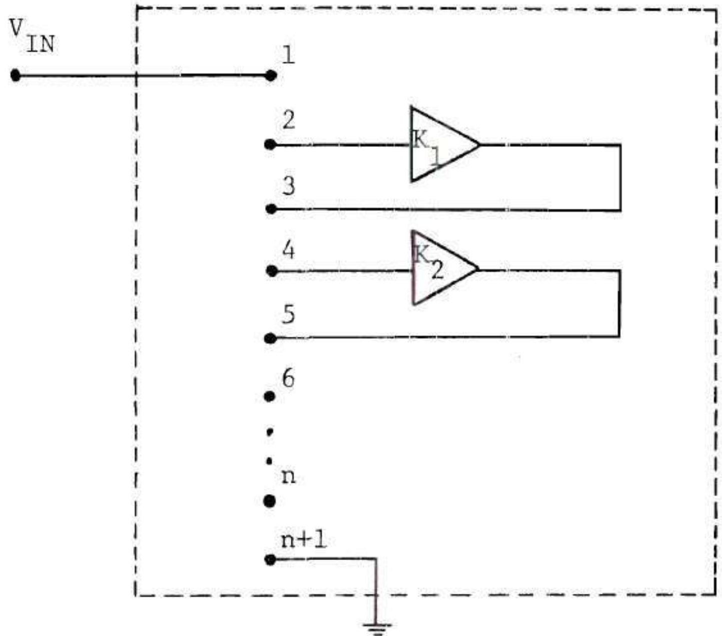


Figure 4. The Configuration #3 of the Two-VCVS Network

The minimum required gains for Figure 3, with $n \geq 5$, is

$$K_{0_{min}} = 2Q_0 \frac{(1+A)}{A} \tag{71}$$

The proof for choices (ii) and (iii) are similar to that for choice (i).

We will consider next Figure 4.

Putting

$$V_3 = K_1 V_2 \tag{72}$$

and

$$V_5 = K_2 V_4 \tag{73}$$

Summing the currents at nodes 2 to n gives the following equation:

$$[\,0\,] = \begin{bmatrix} y_{21} \\ y_{41} \\ y_{61} \\ \cdot \\ \cdot \\ y_{n1} \end{bmatrix} V_1 + \begin{bmatrix} (y_{22}+K_1y_{23}) & (y_{24}+K_2y_{25}) & y_{26} & \cdot & \cdot & y_{2n} \\ (y_{42}+K_1y_{43}) & (y_{44}+K_2y_{45}) & y_{46} & & & y_{4n} \\ (y_{62}+K_1y_{63}) & (y_{64}+K_2y_{65}) & y_{66} & \cdot & \cdot & y_{6n} \\ \cdot\ \cdot\ \cdot\ \cdot & \cdot\ \cdot\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot\ \cdot\ \cdot & \cdot\ \cdot\ \cdot & \cdot & \cdot & \cdot & \cdot \\ (y_{n2}+K_1y_{n3}) & (y_{n4}+K_2y_{n5}) & y_{n6} & \cdot & \cdot & y_{nn} \end{bmatrix} \begin{bmatrix} V_2 \\ V_4 \\ V_6 \\ \cdot \\ \cdot \\ V_n \end{bmatrix} \tag{74}$$

When an output is taken from any node n, $n \geq 2$, the denominator of the voltage-transfer function is

$$D(s) = \begin{vmatrix} (y_{22}+K_1y_{23}) & (y_{24}+K_2y_{25}) & y_{26} & \cdot & \cdot & y_{2n} \\ (y_{42}+K_1y_{43}) & (y_{44}+K_2y_{45}) & y_{26} & \cdot & \cdot & y_{4n} \\ (y_{62}+K_1y_{63}) & (y_{64}+K_2y_{65}) & y_{66} & \cdot & \cdot & y_{6n} \\ \cdot\ \cdot\ \cdot & \cdot\ \cdot\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot\ \cdot\ \cdot & \cdot\ \cdot\ \cdot & \cdot & \cdot & \cdot & \cdot \\ (y_{n2}+K_1y_{n3}) & (y_{n4}+K_2y_{n5}) & y_{n6} & \cdot & \cdot & y_{nn} \end{vmatrix} \tag{75}$$

For $n = 4$,

$$D(s) = y_{44}(y_{22}+K_1y_{23}) - y_{24}(y_{42}+K_1y_{43}) \tag{76}$$

(76) is the function of $(K_1)^0$ only, therefore we cannot have K_0 and K_0^2 terms simultaneously.

For $n = 5$,

$$D(s) = (y_{22}y_{44}-y_{24}y_{42}) + K_1(y_{23}y_{44}-y_{24}y_{43}) \tag{77}$$
$$+K_2(y_{22}y_{45}-y_{25}y_{42}) + K_1K_2(y_{23}y_{45}-{}_{25}y_{43})$$

From (77) with proper values of K's, it is possible to have D(s) in the form of (5) or (9). There are three choices of K's:

(i)
$$K_1 = -K_0 \tag{78}$$

$$K_2 = -K_0 \tag{79}$$

(ii)
$$K_1 = \frac{1}{K_0} \tag{80}$$

$$K_2 = -K_0^2 \tag{81}$$

(iii)
$$K_1 = -K_0^2 \tag{82}$$

$$K_2 = \frac{1}{K_0} \tag{83}$$

The proof for network configuration of Figure 4 will follow that of Figure 3.

If the choice (i) of K's as in (78) and (79) is used, for any n, $n \geq 5$,

$$D(s) = \begin{vmatrix} (y_{22}-K_0y_{23}) & (y_{24}-K_0y_{25}) & y_{26} & \cdot & \cdot & y_{2n} \\ (y_{42}-K_0y_{43}) & (y_{44}-K_0y_{45}) & y_{46} & \cdot & \cdot & y_{4n} \\ (y_{62}-K_0y_{63}) & (y_{64}-K_0y_{65}) & y_{66} & \cdot & \cdot & y_{6n} \\ \cdot\ \cdot\ \cdot & \cdot\ \cdot\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot\ \cdot\ \cdot & \cdot\ \cdot\ \cdot & \cdot & \cdot & \cdot & \cdot \\ (y_{n2}-K_0y_{n3}) & (y_{n4}-K_0y_{n5}) & y_{n6} & & & y_{nn} \end{vmatrix}$$

$$D(s) = \begin{vmatrix} y_{22} & y_{24} & y_{26} & \cdots & y_{2n} \\ y_{42} & y_{44} & y_{46} & \cdots & y_{4n} \\ y_{62} & y_{64} & y_{66} & \cdots & y_{6n} \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ y_{n2} & y_{n4} & y_{n6} & \cdots & y_{nn} \end{vmatrix} - K_0 \begin{vmatrix} y_{23} & y_{24} & y_{26} & \cdots & y_{2n} \\ y_{43} & y_{44} & y_{46} & \cdots & y_{4n} \\ y_{63} & y_{64} & y_{66} & \cdots & y_{6n} \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ y_{n3} & y_{n4} & y_{n6} & \cdots & y_{nn} \end{vmatrix}$$

$$-K_0 \begin{vmatrix} y_{22} & y_{25} & y_{26} & \cdots & y_{2n} \\ y_{42} & y_{45} & y_{46} & \cdots & y_{4n} \\ y_{62} & y_{65} & y_{66} & \cdots & y_{6n} \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ y_{n2} & y_{n5} & y_{n6} & \cdots & y_{nn} \end{vmatrix} + K_0^2 \begin{vmatrix} y_{23} & y_{25} & y_{26} & \cdots & y_{2n} \\ y_{43} & y_{45} & y_{46} & \cdots & y_{4n} \\ y_{63} & y_{65} & y_{66} & \cdots & y_{6n} \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ y_{n3} & y_{n5} & y_{n6} & \cdots & y_{nn} \end{vmatrix} \tag{84}$$

$$D(s) = D_1(s)+K_0D_2(s)+K_0^2D_3(s) \tag{85}$$

$$D_1(s) = \begin{vmatrix} y_{22} & y_{24} & y_{26} & \cdots & y_{2n} \\ y_{42} & y_{44} & y_{46} & \cdots & y_{4n} \\ y_{62} & y_{64} & y_{66} & \cdots & y_{6n} \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ y_{n2} & y_{n4} & y_{n6} & \cdots & y_{nn} \end{vmatrix} \tag{86}$$

$$D_2(s) = -\begin{vmatrix} y_{23} & y_{24} & y_{26} & \cdots & y_{2n} \\ y_{43} & y_{44} & y_{46} & \cdots & y_{4n} \\ y_{63} & y_{64} & y_{66} & \cdots & y_{6n} \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ y_{n3} & y_{n4} & y_{n6} & \cdots & y_{nn} \end{vmatrix} - \begin{vmatrix} y_{22} & y_{25} & y_{26} & \cdots & y_{2n} \\ y_{42} & y_{45} & y_{46} & \cdots & y_{4n} \\ y_{62} & y_{65} & y_{66} & \cdots & y_{6n} \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ y_{n2} & y_{n5} & y_{n6} & \cdots & y_{nn} \end{vmatrix} \tag{87}$$

$$D_3(s) = \begin{vmatrix} y_{23} & y_{25} & y_{26} & \cdots & y_{2n} \\ y_{43} & y_{45} & y_{46} & \cdots & y_{4n} \\ y_{63} & y_{65} & y_{66} & \cdots & y_{6n} \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ y_{n3} & y_{n5} & y_{n6} & \cdots & y_{nn} \end{vmatrix} \tag{88}$$

For either the voltage-transfer function of (5) or (9), the proof will follow that of Figure 3 with n = 5. In both forms, we need to show that

$$D_3(s) \subset D_1(s) \tag{89}$$

From (86), we see that $D_1(s)$ contains the term

$(y_{22}y_{44}y_{66}y_{77} \cdots y_{nn})$ which includes all terms in $D_2(s)$ and $D_3(s)$. Therefore we always have (89) satisfied.

The minimum required gains for Figure 4 when the first choices of K's is chosen will be the same as that of Figure 3, that is, for $n = 5$,

$$K_{0_{min}} = 2Q_0 \frac{(1+A)}{A} \tag{90}$$

If choice II of K's as in (80) and (81) is used, then for any n, $n \geq 5$,

$$D(s) = \begin{vmatrix} (y_{22}+\frac{1}{K_0}y_{23}) & (y_{24}-K_0^2y_{25}) & y_{26} & \cdots & y_{2n} \\ (y_{42}+\frac{1}{K_0}y_{43}) & (y_{44}-K_0^2y_{45}) & y_{46} & \cdots & y_{4n} \\ (y_{62}+\frac{1}{K_0}y_{63}) & (y_{64}-K_0^2y_{65}) & y_{66} & \cdots & y_{6n} \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ (y_{n2}+\frac{1}{K_0}y_{n3}) & (y_{n4}-K_0^2y_{n5}) & y_{n6} & \cdots & y_{nn} \end{vmatrix} \tag{91}$$

With the assumption that $K_0 >> 1$, (91) can be approximated and rearranged as

$$D(s) = D_1(s)+K_0D_2(s)+K_0^2D_3(s) \tag{92}$$

$$D_1(s) = \begin{vmatrix} y_{22} & y_{24} & y_{26} & \cdots & y_{2n} \\ y_{42} & y_{44} & y_{46} & \cdots & y_{4n} \\ y_{62} & y_{64} & y_{66} & \cdots & y_{6n} \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ y_{n2} & y_{n4} & y_{n6} & \cdots & y_{nn} \end{vmatrix} \tag{93}$$

$$D_2(s) = - \begin{vmatrix} y_{23} & y_{25} & y_{26} & \cdots & y_{2n} \\ y_{43} & y_{45} & y_{46} & \cdots & y_{4n} \\ y_{63} & y_{65} & y_{66} & \cdots & y_{6n} \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ y_{n3} & y_{n5} & y_{n6} & \cdots & y_{nn} \end{vmatrix} \tag{94}$$

$$D_3(s) = - \begin{vmatrix} y_{22} & y_{25} & y_{26} & \cdots & y_{2n} \\ y_{42} & y_{45} & y_{46} & \cdots & y_{4n} \\ y_{62} & y_{65} & y_{66} & \cdots & y_{6n} \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ y_{n2} & y_{n5} & y_{n6} & \cdots & y_{nn} \end{vmatrix} \tag{95}$$

For either the voltage-transfer functions of (5) or (9), the proof will follow that of Figure 3 with n = 5. In both forms, we need to show that

$$D_3(s) \subset D_1(s) \tag{96}$$

From (93), we see that $D_1(s)$ contains the term

$(y_{22}y_{44}y_{66}y_{77} \cdots y_{nn})$ which includes all terms in $D_2(s)$ and $D_3(s)$. Therefore we always have (96) satisfied.

The minimum required gains for Figure 4 when choice (ii) of K's is used will be the same as that for choice (i). When choice (iii) of K's is used we will have the same circuit as the choice (ii) with K_1 and K_2 interchanged.

We have shown all possible configurations for circuit of two VCVS and flexible number of nodes, the minimum number of nodes for Figures 3 and 4 is five. When the number of nodes is increased there is no change in the minimum required gains of the VCVS. Therefore it is sufficient to use only five nodes.

Proof of Statement (III)

If three VCVS are given, then D(s) can be realized in one of the three forms:*

(i) $$D(s) = a_0+a_1K_0s+a_2K_0^2s^2 \tag{97}$$

or $$D(s) = a_0'K_0^2+a_1'K_0s+a_2's^2 \tag{98}$$

(ii) $$D(s) = b_0+b_1K_0s+b_2K_0^2s^2+b_3K_0^3s^3 \tag{99}$$

or $$D(s) = b_0'K_0^3+b_1'K_0^2s+b_2'K_0s^2+b_3's^3 \tag{100}$$

(iii) $$D(s) = (c_0+c_1K_0)+c_2K_0^2s+c_3K_0^3s^2 \tag{101}$$

---

*The derivation of these expressions are given in Appendix A.

$$\text{or} \quad D(s) = c_0'K_0^3+c_1'K_0^2s+(c'_2K_0+c_3')s^2 \tag{102}$$

If D(s) is chosen to be of the form as in (97), we have already found that two VCVS can be used to synthesize D(s). Therefore it is not necessary to use three VCVS. One example of this would be the use of two VCVS of gains $\sqrt{K_0}$ connected in cascade to give the equivalent of one VCVS of gain K_0.

If D(s) is chosen to be of the form as in (99), then D(s) is of third order polynomials of s in which we are not interested.

If D(s) is chosen to be of the form as in (101), then one additional approximation has to be made. That is, approximating the coefficient of s^0-term by the term containing K_0only.

$$\begin{aligned} D(s) &\simeq c_1K_0+c_2K_0^2s+c_3K_0^3s^2 \\ &\simeq K_0(c_1+c_2K_0s+c_3K_0^2s^2) \end{aligned} \tag{103}$$

We found that (103) is similar to (97) except for the factor of K_0. This common factor K_0 has no effect on the tuning of the center frequency and at the same time keeping Q_0 constant. The only effect that this common factor K_0 has is the reducing of the gain of the voltage-transfer function. Therefore, there is no need to use three VCVS since we can realize (97) with only two VCVS.

If D(s) is chosen to be of the forms as in (98), (100), or (102), the analysis is similar to that of (97), (99), or (101), respectively.

It can be shown in general that there is no advantage of using more than the necessary two VCVS.

CHAPTER III

DERIVATION OF BAND-PASS FILTER CIRCUITS

In this chapter we shall make a detailed study of second-order

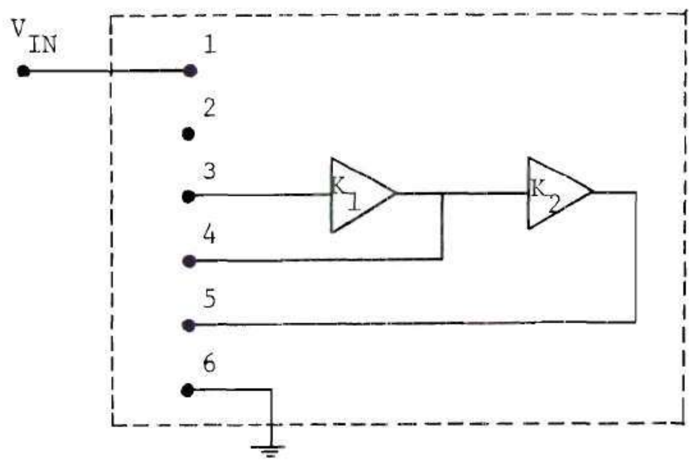


Figure 5. The Configuration #2 of the Two-VCVS Network with n=5

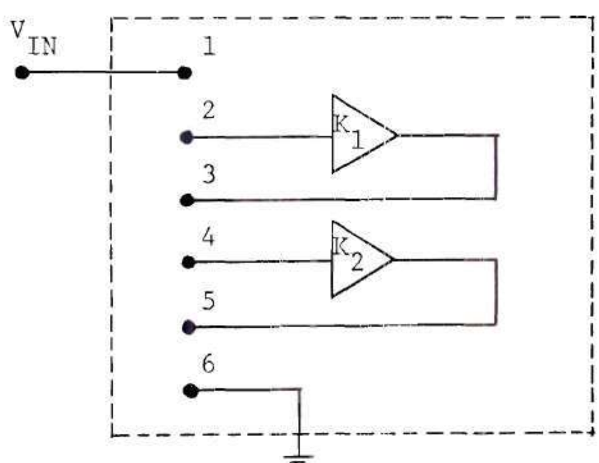


Figure 6. The Configuration #3 of the Two-VCVS Network with n=5

band-pass filter sections based on the method developed in Chapter II. We shall synthesize these filter sections from the basic network configurations shown in Figures 5 and 6, which are the special cases of Figures 3 and 4 with $n = 5$.

Band-pass Filters Based on the Configuration of Figure 5

For the network configuration of Figure 5, the denominator of the voltage-transfer function is

$$D(s) = (y_{22}y_{33}-y_{23}y_{32}) + K_1(y_{22}y_{34}-y_{24}y_{32}) + K_1K_2(y_{22}y_{35}-y_{25}y_{32}) \tag{104}$$

The constant-Q, tunable active band-pass filter can be obtained by having either terminal 3* (or equivalently, 4 or 5) be the output; or terminal 2 be the output.

Filters Obtained by Using Terminal 3 as Output

If the output is to be taken from terminal 3, the numerator of the voltage-transfer function is

---

*If the output is taken from terminal 4,

$$N(s) = K_1(y_{21}y_{32}-y_{22}y_{31})$$

If the output is taken from terminal 5,

$$N(s) = K_1K_2(y_{21}y_{32}-y_{22}y_{31})$$

In either case the type (BP.,LP., or HP.) of the filter remains unchanged.

$$N(s) = y_{21}y_{32}-y_{22}y_{31} \tag{105}$$

for this configuration to be a band-pass section, N(s) must be the function of the first-order of s only. Therefore, we must have

$$y_{31} = 0 \tag{106}$$

Thus
$$N(s) = y_{21}y_{32} \tag{107}$$

(i) If we choose $K_1 = -K_0$ and $K_2 = K_0$, then

$$D(s) = (y_{22}y_{33}-y_{23}y_{32}) \tag{108}$$

$$-K_0(y_{22}y_{34}-y_{24}y_{32})$$

$$-K_0^2(y_{22}y_{35}-y_{25}y_{32})$$

Equations (107) and (108) will be the basic equations for realizing the filters. After searching all possible combinations* of y's, eight circuits as shown in Figures 7 to 14 are found. However, it is most convenient to present these circuits starting with two basic circuits of Figures 7 and 8.

---

*The number of admittances between each node and ground is kept at the minimum. If we remove this restriction, the number of possible circuits can be increased indefinitely. For example, any admittance may be connected between the output of the VCVS and ground without altering the performance of the filter.

For the circuit of Figure 7:

$$N(s) = G_1C_1s \tag{109}$$

$$D(s) = s^2\{C_1C_2(1-K_1K_2) + (C_1C_3+C_2C_3)(1-K_1)\} + s\{G_2(C_1+C_2)$$

$$(110)$$

$$+ (G_1+G_3)(C_1+C_3) - K_1C_3(G_1+G_3) - K_1C_1G_3\} + G_2(G_1+G_3)$$

$$= s^2\{C_1C_2(1+K_0^2) + (C_1C_3+C_2C_3)(1+K_0)\} + s\{G_2(C_1+C_2)$$

$$+ (G_1+G_3)(C_1+C_3) + K_0(C_1G_3+C_3G_1+C_3G_3)\} + G_2(G_1+G_3)$$

For the circuit of Figure 8:

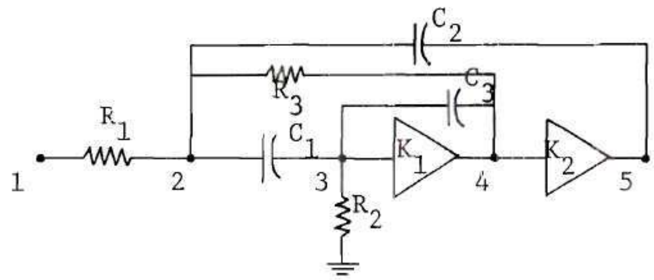
$$N(s) = G_1C_1s \tag{111}$$

$$D(s) = s^2\{C_1C_4(1-K_1K_2) + (1-K_1)(C_1C_2+C_1C_3+C_2C_3+C_2C_4)\} \tag{112}$$

$$+ s\{G_1(C_1+C_2) + G_2(C_1+C_3+C_4)-K_1G_1C_2\} + G_1G_2$$

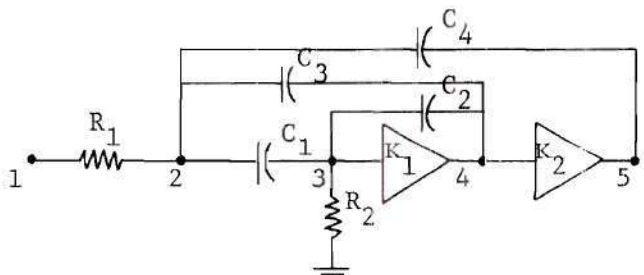
$$= s^2\{C_1C_4(1+K_0^2) + (1+K_0)(C_1C_2+C_1C_3+C_2C_3+C_2C_4)\}$$

$$+ s\{G_1(C_1+C_2) + G_2(C_1+C_3+C_4)+K_0G_1C_2\} + G_1G_2$$



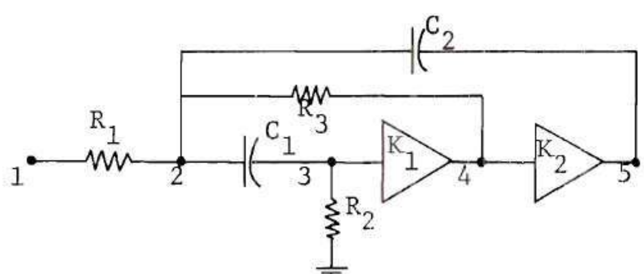
$K_1 = -K_0$
$K_2 = K_0$

Figure 7. A Band-pass Filter Circuit



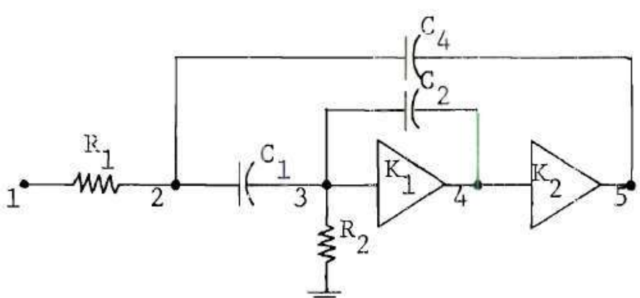
$K_1 = -K_0$
$K_2 = K_0$

Figure 8. A Band-pass Filter Circuit



$K_1 = -K_0$
$K_2 = K_0$

Figure 9. A Band-pass Filter Circuit



$K_1 = -K_0$
$K_2 = K_0$

Figure 10. A Band-pass Filter Circuit

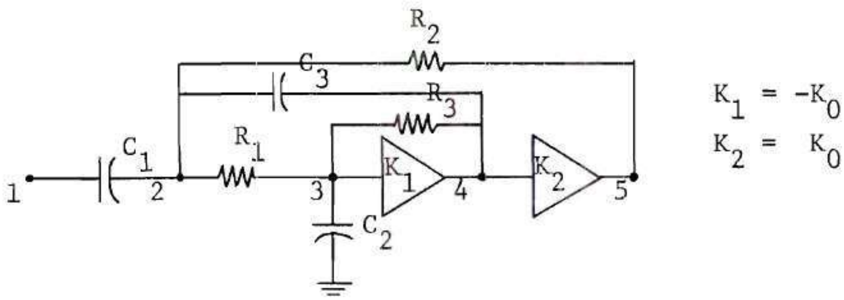


Figure 11. A Band-pass Filter Circuit

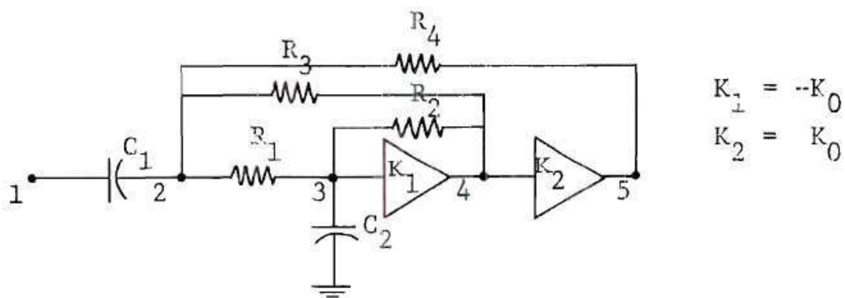


Figure 12. A Band-pass Filter Circuit

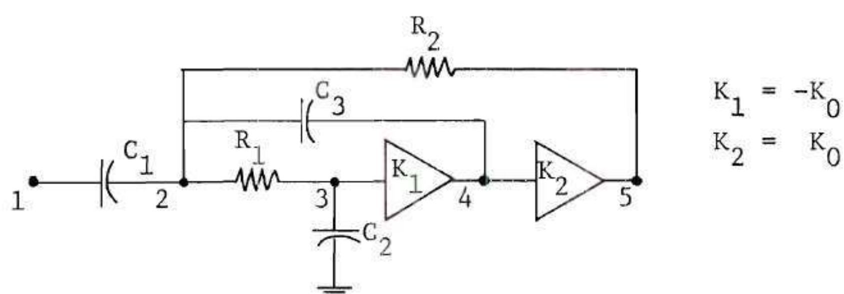


Figure 13. A Band-pass Filter Circuit

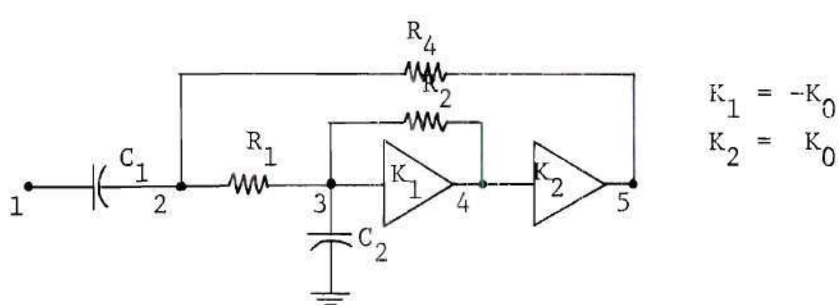


Figure 14. A Band-pass Filter Circuit

For the circuit of Figure 9, N(s) and D(s) will be the same as in (109) and (110) of Figure 7 and C_3 set equal to zero.

For the circuit of Figure 10, N(s) and D(s) will be the same as in (111) and (112) of Figure 8 with C_3 set equal to zero.

It was also found that the duals* of Figures, 7,8,9, and 10 also have the characteristics of constant-Q, tunable band-pass sections. These dual circuits are shown in Figures 11, 12, 13, and 14, respectively.

For the circuit of Figure 11:

$$N(s) = G_1C_1s \tag{113}$$

$$D(s) = s^2C_2(C_1+C_3) + s\{C_2(G_1+G_2)+(C_1+C_3)(G_1+G_3)+K_0 \\ (G_1C_3+G_3C_1+G_3C_3)\} + G_1G_2(1+K_0^2) + (G_1G_3+G_2G_3)(1+K_0) \tag{114}$$

For the circuit of Figure 12:

$$N(s) = G_1C_1s \tag{115}$$

$$D(s) = s^2C_1C_2 + s\{C_1(G_1+G_2)+C_2(G_1+G_3+G_4)+K_0C_1G_2\} \\ + G_1G_4(1+K_0^2)+(1+K_0)(G_1G_2+G_1G_3+G_2G_3+G_2G_4) \tag{116}$$

For the circuit of Figure 13, N(s) and D(s) will be the same as in (113) and (114) of Figure 11 with G_3 set equal to zero.

---

*By duality, we mean $R \leftarrow C$ and $C \leftarrow R$.

For the circuit of Figure 14, N(s) and D(s) will be the same as in (115) and (116) of Figure 12 with G_3 set equal to zero.

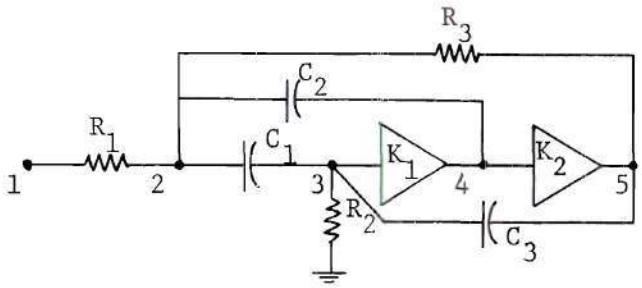
(ii) If we choose $K_1 = -K_0^2$ and $K_2 = \frac{1}{K_0}$, then

$$D(s) = (y_{22}y_{33}-y_{23}y_{32}) \tag{117}$$
$$-K_0(y_{22}y_{35}-y_{25}y_{32})$$
$$-K^2(y_{22}y_{34}-y_{24}y_{32})$$

Equations (107) and (117) will be the basic equations for realizing the filters. After searching all possible combinations of y's, eight circuits of Figures 15 to 22 are found. However, Figures 15 to 22 are similar to Figures 7 to 14, with the interchanging of passive elements connected to nodes 4 and 5. The voltage-transfer functions of Figures 15 to 22 are identical to those of Figures 7 to 14, respectively, after the substitution of values of K_1 and K_2.

(iii) If the network configuration is modified such that $V_4 = K_1V_3$ and $V_5 = K_2V_3$. Then another choice of K's would be $K_1 = -K_0$ and $K_2 = -K_0^2$. We have

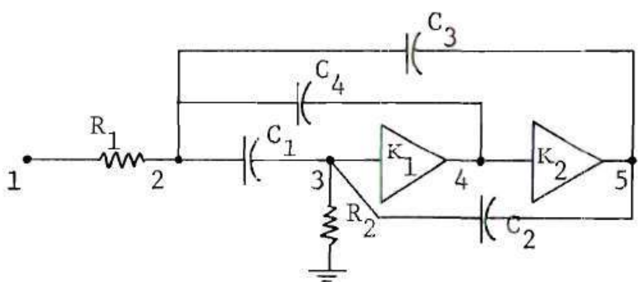
$$D(s) = (y_{22}y_{33}-y_{23}y_{32}) \tag{118}$$
$$-K_0(y_{22}y_{34}-y_{24}y_{32})$$
$$-K_0^2(y_{22}y_{35}-y_{25}y_{32})$$



$$K_1 = -K_0^2$$

$$K_2 = \frac{1}{K_0}$$

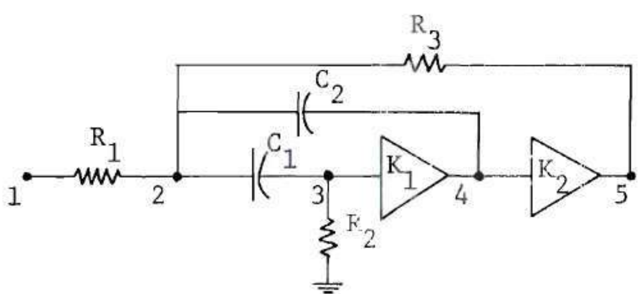
Figure 15. A Band-pass Filter Circuit



$$K_1 = -K_0^2$$

$$K_2 = \frac{1}{K_0}$$

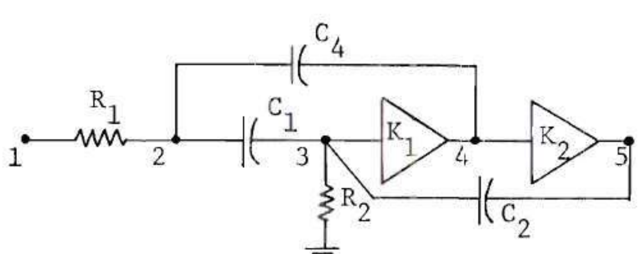
Figure 16. A Band-pass Filter Circuit



$$K_1 = -K_0^2$$

$$K_2 = \frac{1}{K_0}$$

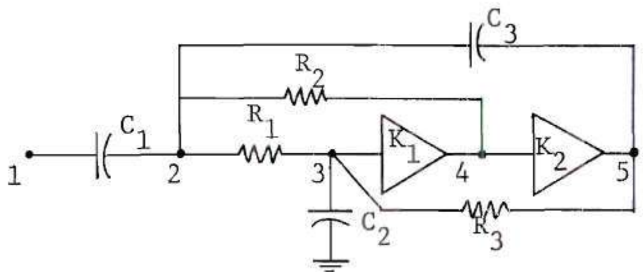
Figure 17. A Band-pass Filter Circuit



$$K_1 = -K_0^2$$

$$K_2 = \frac{1}{K_0}$$

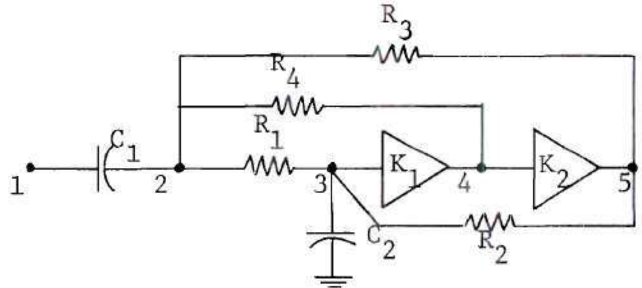
Figure 18. A Band-pass Filter Circuit



$$K_1 = -K_1^2$$

$$K_2 = \frac{1}{K_0}$$

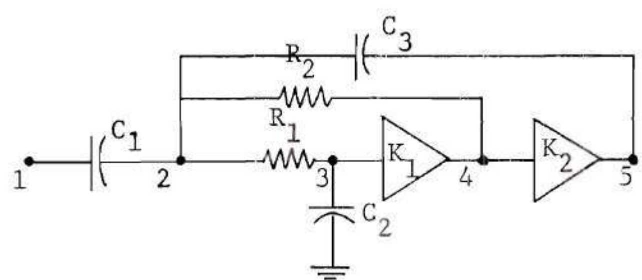
Figure 19. A Band-pass Filter Circuit



$$K_1 = -K_0^2$$

$$K_2 = \frac{1}{K_0}$$

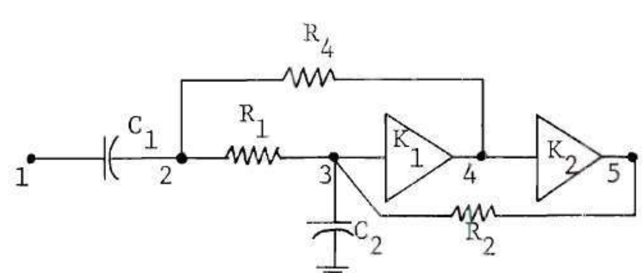
Figure 20. A Band-pass Filter Circuit



$$K_1 = -K_0^2$$

$$K_2 = \frac{1}{K_0}$$

Figure 21. A Band-pass Filter Circuit



$$K_1 = -K_0^2$$

$$K_2 = \frac{1}{K_0}$$

Figure 22. A Band-pass Filter Circuit

Equation (118) is identical to (108). Equations (118) and (107) will be the basic equations for realizing the filters. After searching all possible combinations of y's, eight circuits as shown in Figures 23 to 30 are found. However, these figures are similar to Figures 7 to 14, with changes in the locations of K_2's. The voltage-transfer functions of Figures 23 to 30 are identical to those of Figure 7 to 14, respectively, after the substitution of values of K_1 and K_2.

Filters Obtained by Using Terminal 2 as Output

If the output is to be taken from terminal 2, the numerator of the voltage-transfer function is

$$N(s) = y_{31}(y_{23}+K_1y_{24}+K_1K_2y_{25}) - y_{21}(y_{33}+K_1y_{34}+K_1K_2y_{35}) \tag{119}$$

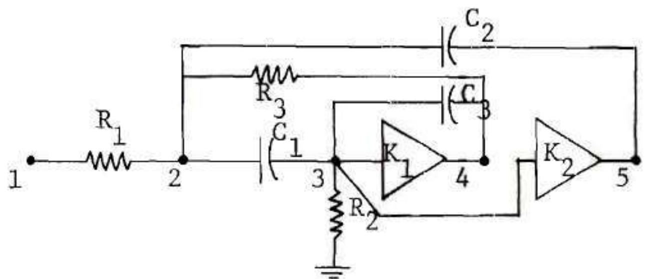
for this configuration to be a band-pass section, N(s) must be the function of the first-order of s only. Therefore, we must have

$$y_{21} = 0 \tag{120}$$

Thus
$$N(s) = y_{31}(y_{23}+K_1y_{24}+K_1K_2y_{25}) \tag{121}$$

The denominator of the voltage-transfer function is identical to (104).

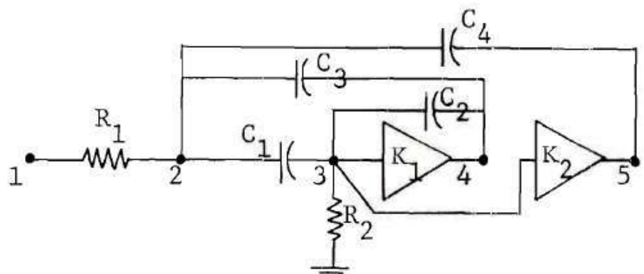
(i) If we choose $K_1 = -K_0$ and $K_2 = K_0$, then D(s) can be expressed as in (108). Equations (108) and (121) will be the basic equations for realizing the filters. After searching all possible combinations of



$K_1 = -K_0$

$K_2 = -K_0^2$

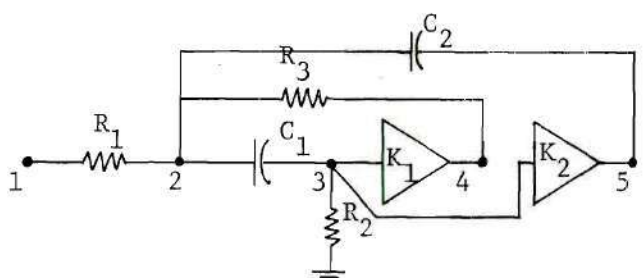
Figure 23. A Band-pass Filter Circuit



$K_1 = -K_0$

$K_2 = -K_0^2$

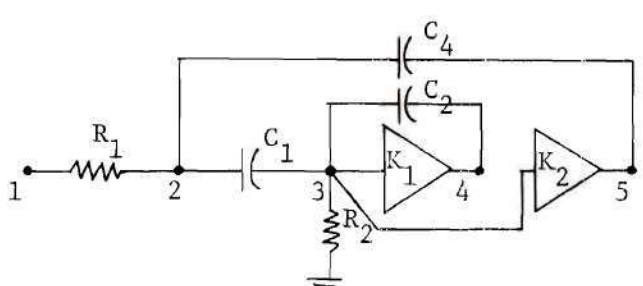
Figure 24. A Band-pass Filter Circuit



$K_1 = -K_0$

$K_2 = -K_0^2$

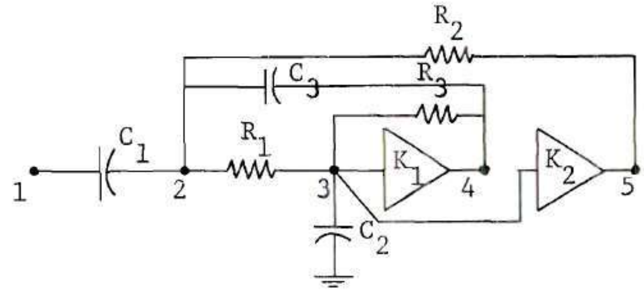
Figure 25. A Band-pass Filter Circuit



$K_1 = - K_0$

$K_2 = -K_0^2$

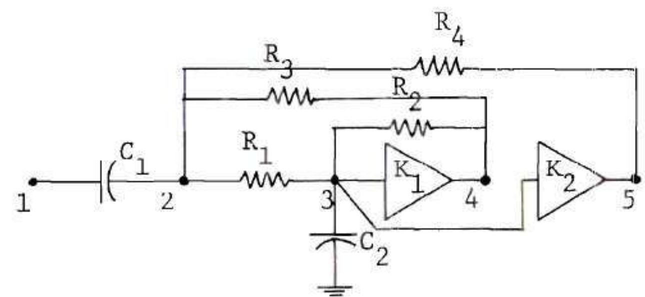
Figure 26. A Band-pass Filter Circuit



$K_1 = -K_0$

$K_2 = -K_0^2$

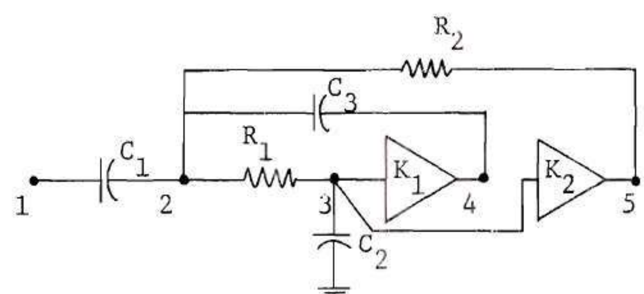
Figure 27. A Band-pass Filter Circuit



$K_1 = -K_0$

$K_2 = -K_0^2$

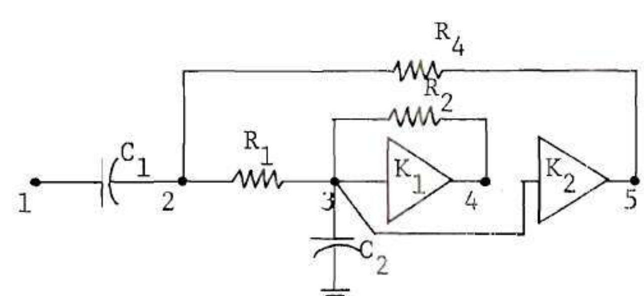
Figure 28. A Band-pass Filter Circuit



$K_1 = -K_0$

$K_2 = -K_0^2$

Figure 29. A Band-pass Filter Circuit



$K_1 = -K_0$

$K_2 = -K_0^2$

Figure 30. A Band-pass Filter Circuit

y's, four circuits as shown in Figures 31 to 34 are found. The first and basic circuit is shown in Figure 31.

For the circuit of Figure 31:

$$N(s) = G_1(C_1+K_1C_4+K_1K_2C_2)s \tag{122}$$

$$= G_1(C_1-K_0C_4-K_0^2C_2)s$$

$$D(s) = s^2\{C_1C_2(1-K_1K_2)+(C_1C_3+C_1C_4+C_2C_3+C_3C_4)(1-K_1)\} \tag{123}$$

$$+s\{G_1(C_1+C_2+C_4)+G_2(C_1+C_3)-K_1G_2C_3\} + G_1G_2$$

$$= s^2\{C_1C_2(1+K_0^2)+(C_1C_3+C_1C_4+C_2C_3+C_3C_4)(1+K_0)\}$$

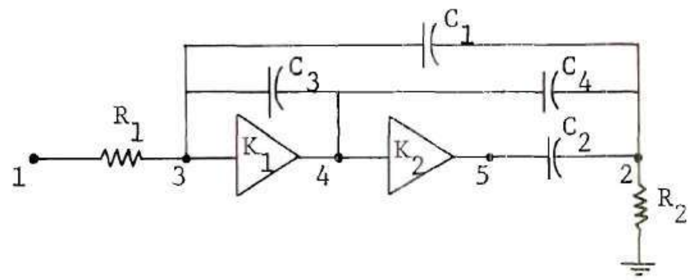
$$+s\{G_1(C_1+C_2+C_4)+G_2(C_1+C_3)+K_0G_2C_3\} + G_1G_2$$

For the circuit of Figure 32, N(s) and D(s) will be the same as in (122) and (123) of Figure 31 with C_4 set equal to zero.

It was also found that the duals of Figures 31 and 32 also have the characteristics of constant-Q, tunable band-pass sections. These dual circuits are shown in Figures 33 and 34, respectively.

For the circuit of Figure 33:

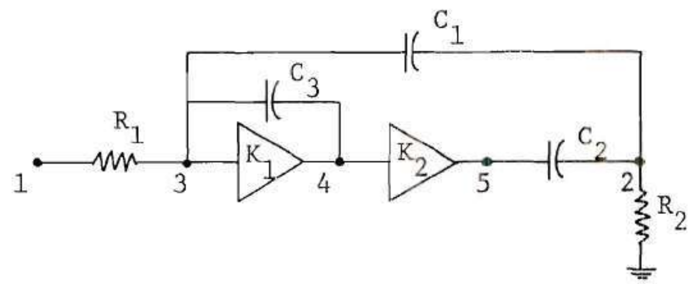
$$N(s) = C_1(G_1-K_0G_4-K_0^2G_2) \tag{124}$$



$K_1 = -K_0$

$K_2 = K_0$

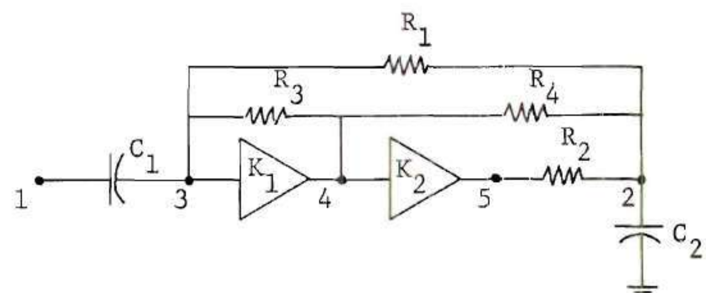
Figure 31. A Band-pass Filter Circuit



$K_1 = -K_0$

$K_2 = K_0$

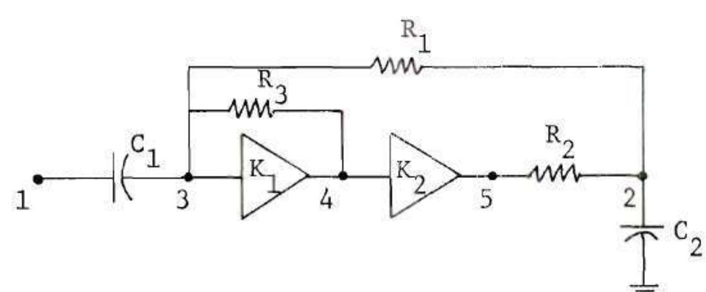
Figure 32. A Band-pass Filter Circuit



$K_1 = -K_0$

$K_2 = K_0$

Figure 33. A Band-pass Filter Circuit



$K_1 = -K_0$

$K_2 = K_0$

Figure 34. A Band-pass Filter Circuit

$$D(s) = s^2C_1C_2+s\{C_1(G_1+G_2+G_4)+C_2(G_1+G_3)+K_0C_2G_3\} \tag{125}$$

$$+G_1G_2(1+K_0^2)+(G_1G_3+G_1G_4+G_2G_3+G_3G_4)(1+K_0)$$

For the circuit of Figure 34, N(s) and D(s) will be the same as in (124) and (125) of Figure 33 with G_4 set equal to zero.

(ii) If we choose $K_1 = -K_0^2$ and $K_2 = \frac{1}{K_0}$, then D(s) can be expressed as in (117), and

$$N(s) = y_{31}(y_{23}-K_0y_{25}-K_0^2y_{24}) \tag{126}$$

Equations (117) and (126) will be the basic equations for realizing the filters. After searching all possible combinations of y's, four circuits as shown in Figures 35 to 38 are found. However, these figures are similar to Figures 31 to 34, with the interchanging of passive elements connected to nodes 4 and 5. The voltage-transfer functions of Figures 35 to 38 are identical to those of Figures 31 to 34, respectively, after the substitution of values of K_1 and K_2.

(iii) If the network configuration is modified such that $V_4 = K_1V_3$ and $V_5 = K_2V_3$. Then another choice of K's would be $K_1 = -K_0$ and $K_2 = -K_0^2$. We have D(s) as in (118), and

$$N(s) = y_{31}(y_{23}-K_0y_{24}-K_0^2y_{25}) \tag{127}$$

Equations (118) and (127) are the basic equations for realizing the filters. After searching all possible combinations of y's, four circuits as shown in Figures 39 to 42 are found. However, these figures are similar to Figures 31 to 34, with changes in the locations of K_2's. The voltage-transfer functions of Figures 39 to 42 are identical to those of Figures 31 to 34, respectively, after the substitution of values of K_1 and K_2.

Band-pass Filters Based on the Configuration of Figure 6

If the network configuration of Figure 6 is used, the denominator of the voltage-transfer function is

$$D(s) = (y_{22}y_{44}-y_{24}y_{42})+K_1(y_{23}y_{44}-y_{24}y_{43}) \tag{128}$$

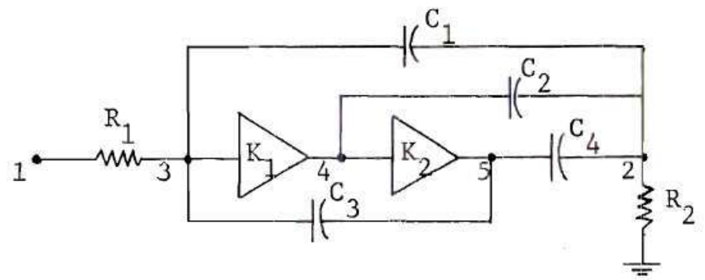
$$+K_2(y_{22}y_{45}-y_{25}y_{42})+K_1K_2(y_{23}y_{45}-y_{25}y_{43})$$

The output may be taken from terminal 2, 3, 4, or 5. However, it should become apparent that these options are all basically equivalent to one another. If the output is to be taken from node 4, then numerator of the voltage-transfer function will be

$$N(s) = y_{21}(y_{42}+K_1y_{43})-y_{41}(y_{22}+K_1y_{23}) \tag{129}$$

For this configuration to be a band-pass section, N(s) must be the function of the first-order of s only. Therefore, we must have

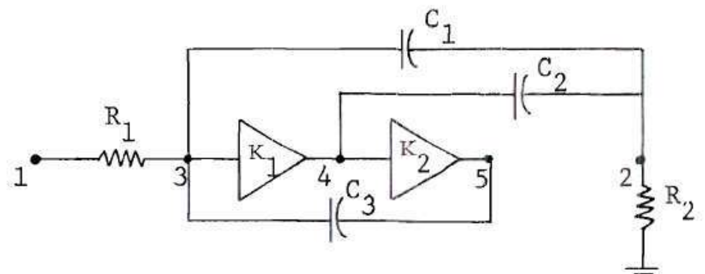
$$y_{41} = 0 \tag{130}$$



$K_1 = -K_0^2$

$K_2 = \frac{1}{K_0}$

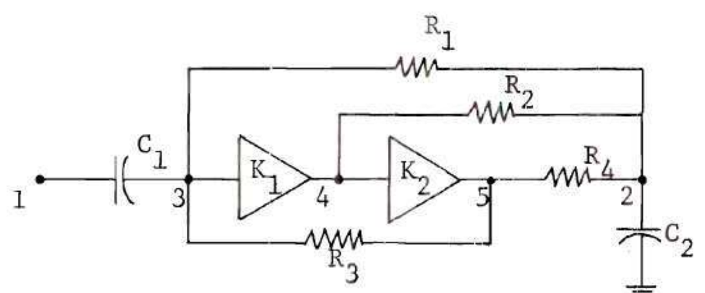
Figure 35. A Band-pass Filter Circuit



$K_1 = -K_0^2$

$K_2 = \frac{1}{K_0}$

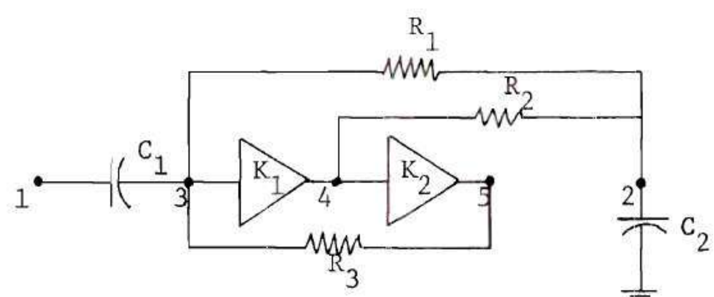
Figure 36. A Band-pass Filter Circuit



$K_1 = -K_0^2$

$K_2 = \frac{1}{K_0}$

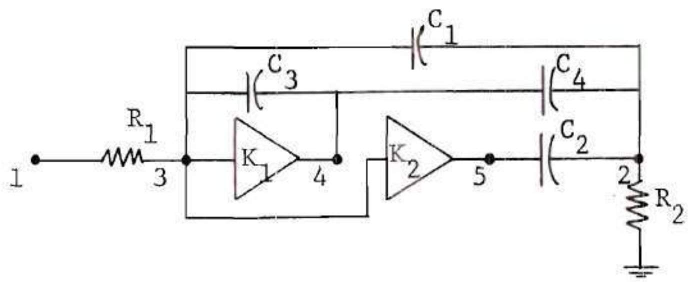
Figure 37. A Band-pass Filter Circuit



$K_1 = -K_0^2$

$K_2 = \frac{1}{K_0}$

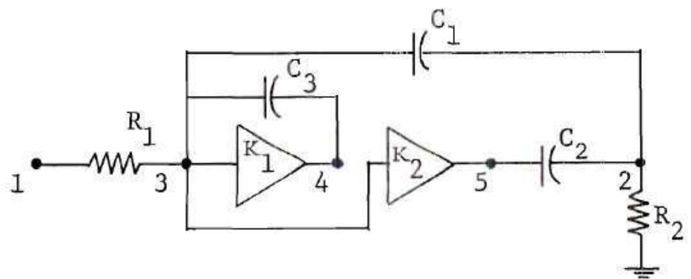
Figure 38. A Band-pass Filter Circuit



$$K_1 = -K_0$$

$$K_2 = -K_0^2$$

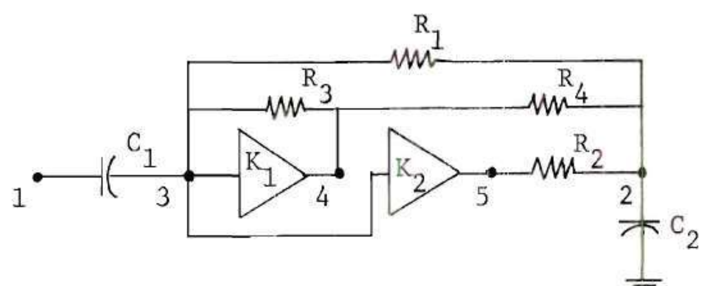
Figure 39. A Band-pass Filter Circuit



$$K_1 = -K_0$$

$$K_2 = -K_0^2$$

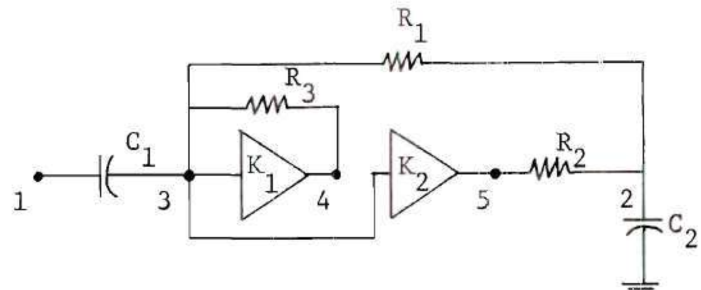
Figure 40. A Band-pass Filter Circuit



$$K_1 = -K_0$$

$$K_2 = -K_0^2$$

Figure 41. A Band-pass Filter Circuit



$$K_1 = -K_0$$

$$K_2 = -K_0^2$$

Figure 42. A Band-pass Filter Circuit

Thus

$$N(s) = y_{21}(y_{42}+K_1y_{43}) \tag{131}$$

(i) If we choose $K_1 = -K_0$ and $K_2 = -K_0$, then from (128) and (131)

$$D(s) = (y_{22}y_{44}-y_{24}y_{42})-K_0(y_{23}y_{44}-y_{24}y_{43} \tag{132}$$

$$+y_{22}y_{45}-y_{25}y_{42})+K_0^2(y_{23}y_{45}-y_{25}y_{43})$$

$$N(s) = y_{21}(y_{42}-K_0y_{43}) \tag{133}$$

Equations (132) and (133) will be the basic equations for realizing the filters. After searching all possible combinations of y's, eighteen circuits as shown in Figures 43 to 60 are found. However, it is most convenient to present Figures 43 to 48 by Table 1 and their duals, Figures 49 to 54, by Table 2. These tables show the basic circuits of each group and show which resistors or capacitors are retained in each figure. For example, Figure 44 is derived from Figure 43 with C_1 deleted.

For the circuit of Figure 43:

$$N(s) = G_1(C_4+K_1C_1)s \tag{134}$$

$$= G_1(C_4-K_0C_1)s$$

$$D(s) = s^2\{C_2C_3(1-K_1-K_2+K_1K_2)+C_1C_5(1-K_1K_2) \tag{135}$$

$$+(C_1C_2+C_2C_4+C_1C_4)(1-K_1)+(C_3C_4+C_3C_5+C_4C_5)(1-K_2)\}$$

$$+s(G_1(C_1+C_3+C_4)+G_2(C_2+C_4+C_5)-K_1G_2C_2-K_2G_1C_3)+G_1G_2$$

$$= \quad s^2\{C_2C_3(1+2K_0+K_0^2)+C_1C_5(1-K_0^2)$$

$$+(C_1C_2+C_1C_4+C_2C_4+C_3C_4+C_3C_5+C_4C_5)(1+K_0)\}$$

$$+s\{G_1(C_1+C_3+C_4)+G_2(C_2+C_4+C_5)+K_0(G_2C_2+G_1C_3)\}+G_1G_2$$

For the circuits of Figures 44 to 48, the voltage-transfer functions can be found by using equations (134), (135), and Table 1. For example, N(s) and D(s) of Figure 44 are those of Figure 43 with C_1 set equal to zero.

For the circuit of Figure 49:

$$N(s) \quad = \quad C_1(G_4-K_0G_1)s \tag{136}$$

$$D(s) \quad = \quad s^2C_1C_2+s\{C_1(G_1+G_3+G_4)+C_2(G_2+G_4+G_5) \tag{137}$$

$$+K_0(C_2G_2+C_1G_3)\}+G_2G_3(1+2K_0+K_0^2)+G_1G_5(1-K_0^2)$$

$$+(C_1C_2+C_1C_4+C_2C_4+C_3C_4+C_3C_5+C_4C_5)(1+K_0)$$

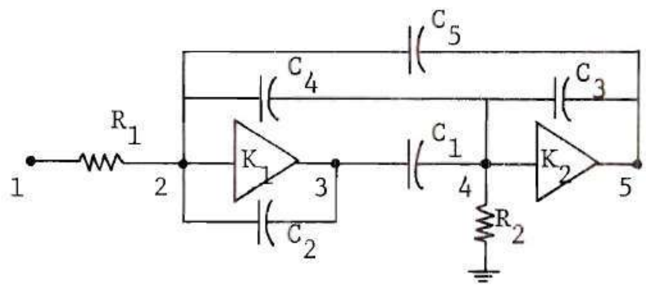
For the circuits of Figures 50 to 54, the voltage-transfer functions

Table 1. The Relationships of Circuits of Figures 43 to 48

Figure	C_1	C_2	C_3	C_4	C_5
43	1	1	1	1	1
44	0	1	1	1	1
45	1	1	1	0	1
46	1	1	1	1	0
47	1	1	1	0	0
48	0	1	1	1	0

Table 2. The Relationships of Circuits of Figures 49 to 54

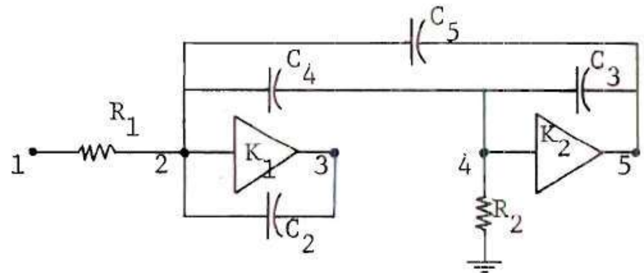
Figure	R_1	R_2	R_3	R_4	R_5
49	1	1	1	1	1
50	0	1	1	1	1
51	1	1	1	0	1
52	1	1	1	1	0
53	1	1	1	0	0
54	0	1	1	1	0



$K_1 = -K_0$

$K_2 = -K_0$

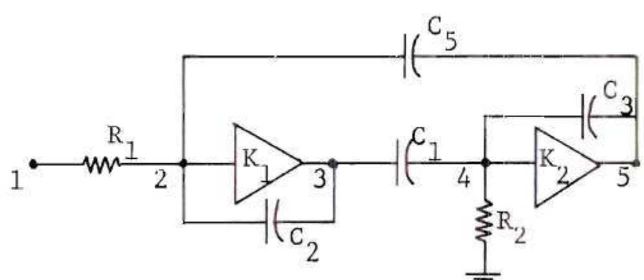
Figure 43. A Band-pass Filter Circuit



$K_1 = -K_0$

$K_2 = -K_0$

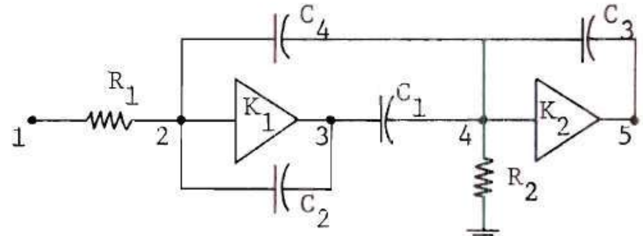
Figure 44. A Band-pass Filter Circuit



$K_1 = -K_0$

$K_2 = -K_0$

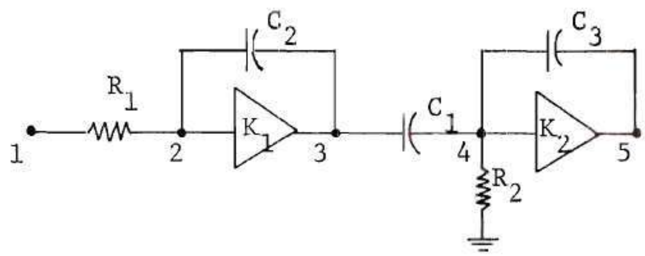
Figure 45. A Band-pass Filter Circuit



$K_1 = -K_0$

$K_2 = -K_0$

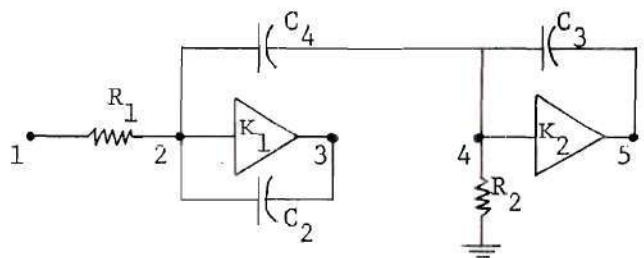
Figure 46. A Band-pass Filter Circuit



$K_1 = -K_0$

$K_2 = -K_0$

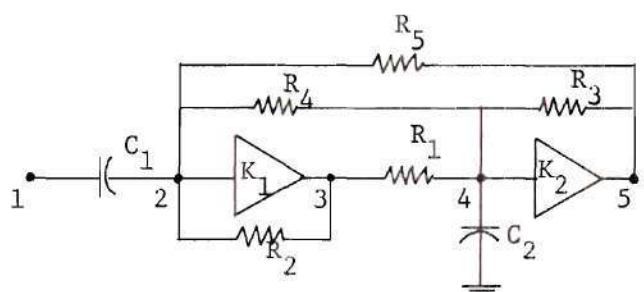
Figure 47. A Band-pass Filter Circuit



$K_1 = -K_0$

$K_2 = -K_0$

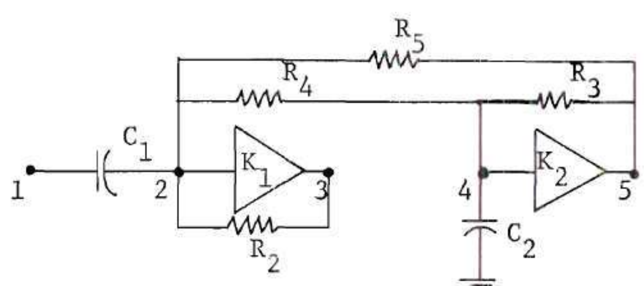
Figure 48. A Band-pass Filter Circuit



$K_1 = -K_0$

$K_2 = -K_0$

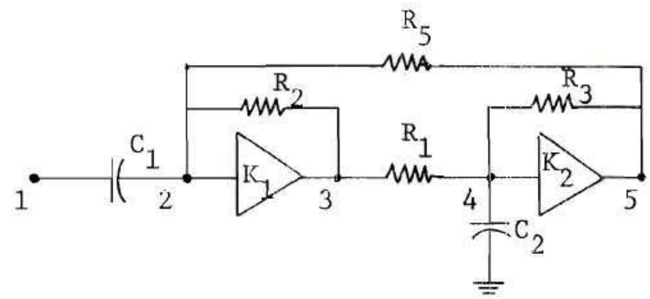
Figure 49. A Band-pass Filter Circuit



$K_1 = -K_0$

$K_2 = -K_0$

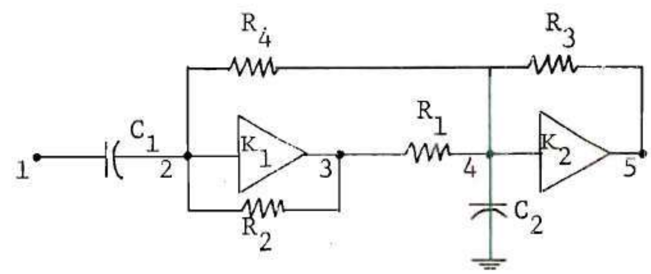
Figure 50. A Band-pass Filter Circuit



$K_1 = -K_0$

$K_2 = -K_0$

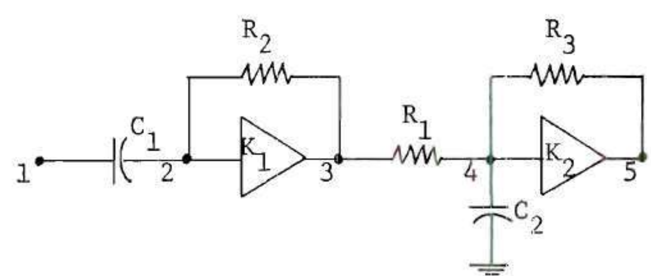
Figure 51. A Band-pass Filter Circuit



$K_1 = -K_0$

$K_2 = -K_0$

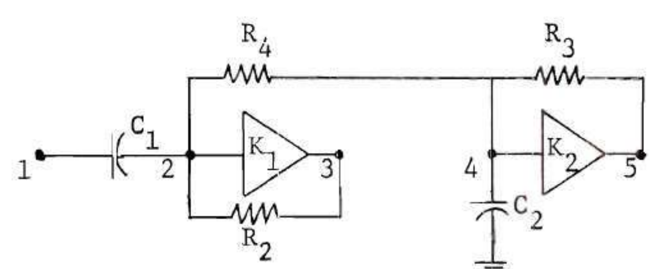
Figure 52. A Band-pass Filter Circuit



$K_1 = -K_0$

$K_2 = -K_0$

Figure 53. A Band-pass Filter Circuit



$K_1 = -K_0$

$K_2 = -K_0$

Figure 54. A Band-pass Filter Circuit

can be found by using equations (136), (137), and Table 2. For example, N(s) and D(s) of Figure 50 are those of Figure 49 with G_1 set equal to zero.

For the circuit of Figure 55:

$$N(s) = G_1C_2K_1s \tag{138}$$

$$= -G_1C_2K_0s$$

$$D(s) = s^2C_1C_2+s\{C_1G_3+C_2(G_1+G_2)-K_1G_2C_2-K_2G_3C_1\} \tag{139}$$

$$+G_1G_3(1-K_2)+G_2G_3(1-K_1-K_2+K_1K_2)$$

$$= s^2C_1C_2+s\{C_1G_3+C_2(G_1+G_2)+K_0(G_2C_2+G_3C_1)\}$$

$$+G_1G_3(1+K_0)+G_2G_3(1+2K_0+K_0^2)$$

For the circuit of Figure 56:

$$N(s) = G_1C_2s \tag{140}$$

$$D(s) = s^2C_1C_2+s\{G_3(C_1+C_2)+(G_1+G_2+G_4)C_2-K_1G_2C_2 \tag{141}$$

$$-K_2G_3(C_1+C_2)-K_2G_4C_2\}+G_2G_3(1-K_1-K_2+K_1K_2)$$

$$+G_3(G_1+G_4)(1-K_2)$$

$$= s^2C_1C_2+s\{G_3(C_1+C_2)+(G_1+G_2+G_4)C_2+K_0(G_2C_2$$

$$+G_3(C_1+C_2)+G_4C_2)\}+G_2G_3(1+2K_0+K_0^2)+G_3(G_1+G_4)(1+K_0)$$

For the circuit of Figure 57 which is derived from Figure 56, N(s) and D(s) will be the same as in (140) and (141) of Figure 56 with G_4 set equal to zero.

For the circuit of Figure 58 which is the dual of Figure 55:

$$N(s) = -C_1G_2K_0s \tag{142}$$

$$D(s) = s^2\{C_1C_3(1+K_0)+C_2C_3(1+2K_0+K_0^2)\} \tag{143}$$

$$+s\{G_1C_3+G_2(C_1+C_2)+K_0(C_2G_2+C_3G_1)\}+G_1G_2$$

For the circuit of Figure 59 which is the dual of Figure 56:

$$N(s) = C_1G_2s \tag{144}$$

$$D(s) = s^2\{C_2C_3(1+2K_0+K_0^2)+C_3(C_1+C_4)(1+K_0)\} \tag{145}$$

$$+s\{C_3(G_1+G_2)+(C_1+C_2+C_4)G_2+K_0(C_2G_2+C_3(G_1+G_2)$$

$$+C_4G_2\}+G_1G_2$$

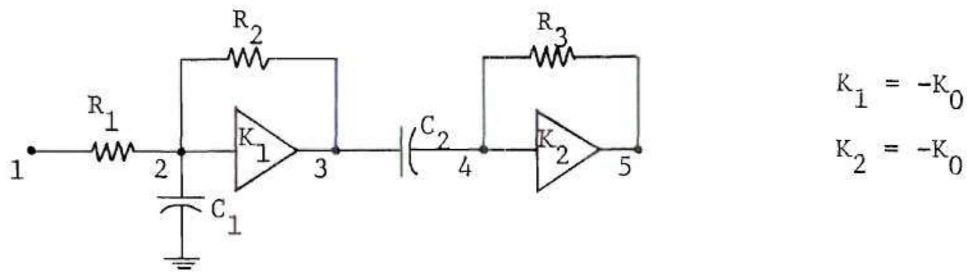


Figure 55. A Band-pass Filter Circuit

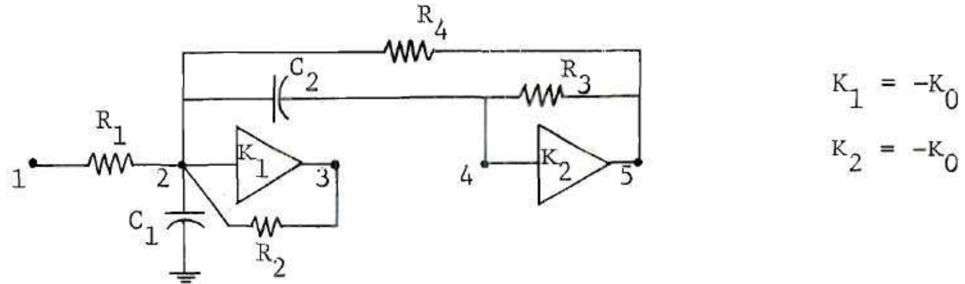


Figure 56. A Band-pass Filter Circuit

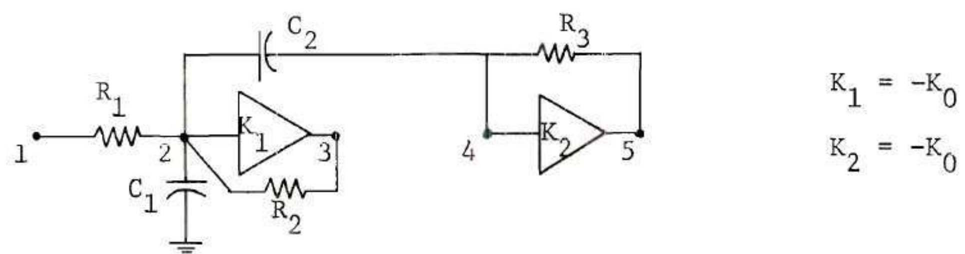


Figure 57. A Band-pass Filter Circuit

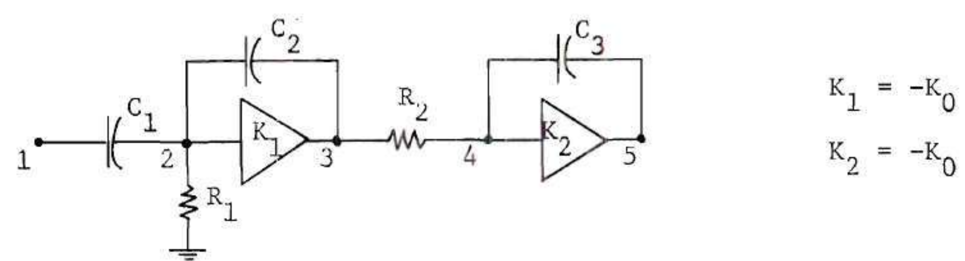


Figure 58. A Band-pass Filter Circuit

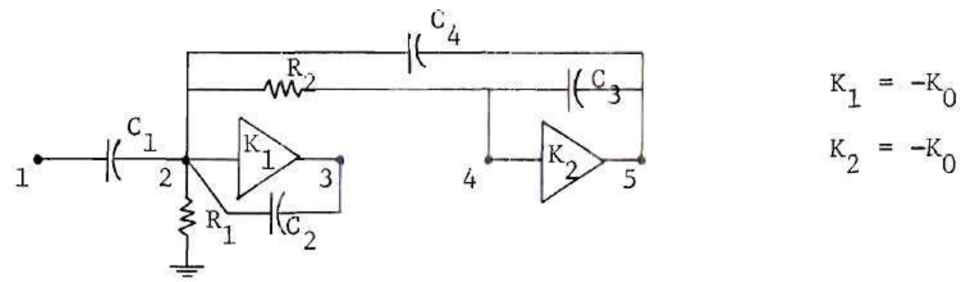


Figure 59. A Band-pass Filter Circuit

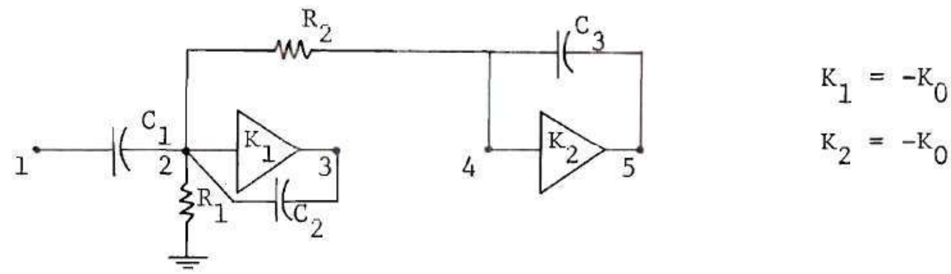


Figure 60. A Band-pass Filter Circuit

For the circuit of Figure 60 which is the dual of Figure 57, N(s) and D(s) will be the same as in (144) and (145) of Figure 59 with C_4 set equal to zero.

(ii) If we choose $K_1 = K_0$ and $K_2 = -K_0$, then from (128) and (131)

$$D(s) = (y_{22}y_{44}-y_{24}y_{42})+K_0(y_{23}y_{44}-y_{24}y_{43}) \tag{146}$$

$$-K_0(y_{22}y_{45}-y_{25}y_{42})-K_0^2(y_{23}y_{45}-y_{25}y_{43})$$

$$N(s) = y_{21}(y_{42}+K_0y_{43}) \tag{147}$$

Equations (146) and (147) will be the basic equations for realizing the filters. After searching all possible combinations of y's, eight circuits as shown in Figures 61 to 68 are found. However, it is most convenient to present Figures 61 to 64 by Table 3 and their duals, Figures 65 to 68, by Table 4. These tables show the basic circuits of each group and show which resistors or capacitors are retained in each figure.

For the circuit of Figure 61:

$$N(s) = G_1(C_4+K_1C_1)s \tag{148}$$

$$= G_1(C_4+K_0C_1)s$$

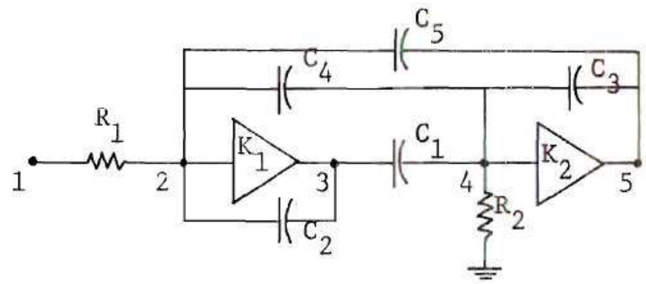
$$D(s) = s^2\{C_2C_3(1+K_1K_2)+C_1C_5(1-K_1K_2) \tag{149}$$

Table 3. The Relationships of Circuits of Figures 61 to 64

Figure	C_1	C_2	C_3	C_4	C_5
61	1	1	1	1	1
62	1	0	1	1	1
63	1	1	1	0	1
64	1	0	1	0	1

Table 4. The Relationships of Circuits of Figures 65 to 68

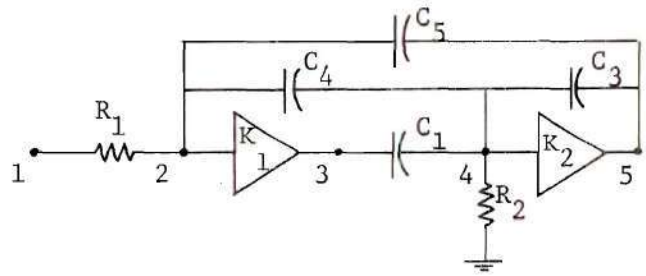
Figure	R_1	R_2	R_3	R_4	R_5
65	1	1	1	1	1
66	1	0	1	1	1
67	1	1	1	0	1
68	1	0	1	0	1



$K_1 = K_0$

$K_2 = -K_0$

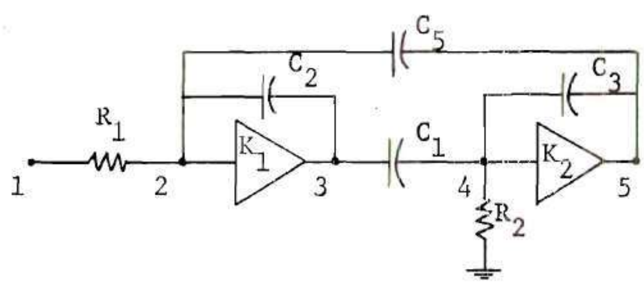
Figure 61. A Band-pass Filter Circuit



$K_1 = K_0$

$K_2 = -K_0$

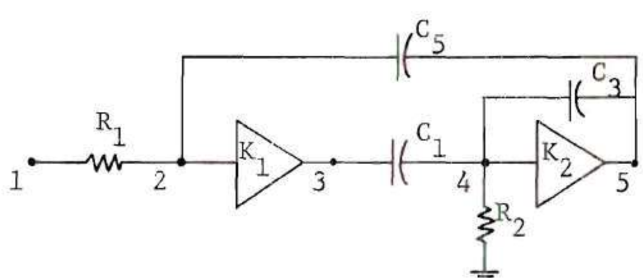
Figure 62. A Band-pass Filter Circuit



$K_1 = K_0$

$K_2 = -K_0$

Figure 63. A Band-pass Filter Circuit



$K_1 = K_0$

$K_2 = -K_0$

Figure 64. A Band-pass Filter Circuit

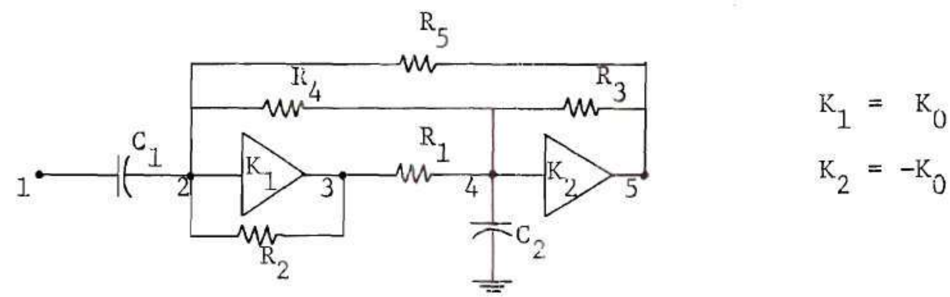


Figure 65. A Band-pass Filter Circuit

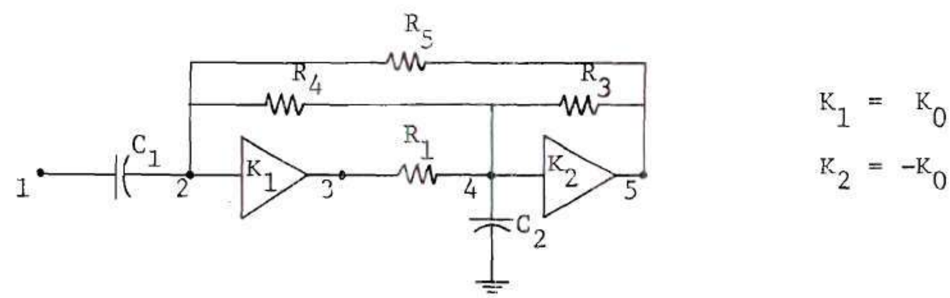


Figure 66. A Band-pass Filter Circuit

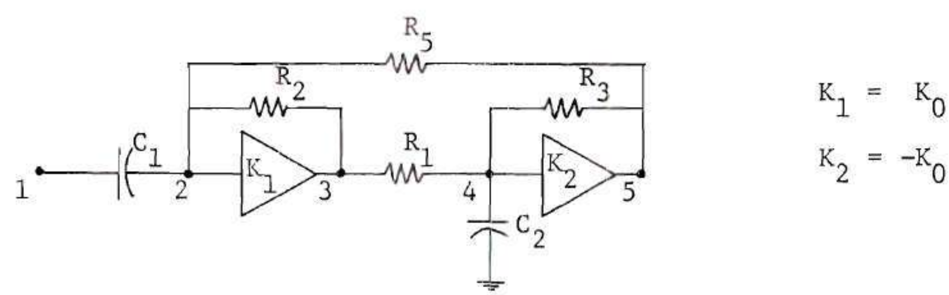


Figure 67. A Band-pass Filter Circuit

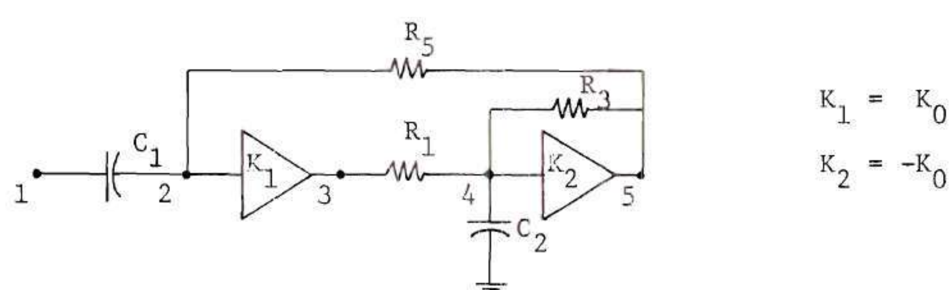


Figure 68. A Band-pass Filter Circuit

$$+(C_1C_2+C_2C_4+C_1C_4)(1-K_1) + (C_3C_4+C_3C_5+C_4C_5)(1-K_2)\}$$

$$+s\{G_1(C_1+C_3+C_4)+G_2(C_2+C_4+C_5)-K_1G_2C_2-K_2G_1C_3\}+G_1G_2$$

$$= \quad s^2\{C_2C_3(1-K_0^2)+C_1C_5(1+K_0^2)+(C_1C_2+C_2C_4+C_1C_4)(1-K_0)$$

$$+(C_3C_4+C_3C_5+C_4C_5)(1+K_0)\}+s\{G_1(C_1+C_3+C_4)$$

$$+G_2(C_2+C_4+C_5)+K_0(-G_2C_2+G_1C_3)\}+G_1G_2$$

For the circuits of Figures 62 to 64, the voltage-transfer functions can be found by using equations (148), (149), and Table 3, for example, N(s) and D(s) of Figure 62 are those of Figure 61 with C_2 set equal to zero.

For the circuit of Figure 65:

$$N(s) \quad = \quad C_1(G_4+K_0G_1)s \tag{150}$$

$$D(s) \quad = \quad s^2C_1C_2+s\{C_1(G_1+G_3+G_4)+C_2(G_2+G_4+G_5) \tag{151}$$

$$+K_0(-C_2G_2+C_1G_3)\}+G_2G_3(1-K_0^2)+G_1G_5(1+K_0^2)$$

$$+(G_1G_2+G_2G_4+G_1G_4)(1-K_0)+(G_3G_4+G_3G_5+G_4G_5)(1+K_0)$$

For the circuits of Figures 66 to 68, the voltage-transfer functions can be found by using equations (150), (151), and Table 4, for example, N(s) and D(s) of Figure 66 are those of Figure 65 with G_2 set equal to zero.

(iii) If we choose $K_1 = -K_0$ and $K_2 = K_0$, then from (128) and (131)

$$D(s) = (y_{22}y_{44}-y_{24}y_{42})-K_0(y_{23}y_{44}-y_{24}y_{43}) \tag{152}$$

$$+K_0(y_{22}y_{45}-y_{25}y_{42})-K_0^2(y_{23}y_{45}-y_{25}y_{43})$$

$$N(s) = y_{21}(y_{42}-K_0y_{43}) \tag{153}$$

Equations (152) and (153) will be the basic equations for realizing the filters. After searching all possible combinations of y's, eight circuits as shown in Figures 69 to 76 are found. However, it is most convenient to present Figures 69 to 72 by Table 5 and their duals, Figures 73 to 76, by Table 6. These tables show the basic circuits of each group and show which resistors or capacitors are retained in each figure.

For the circuit of Figure 69:

$$N(s) = G_1(C_4+K_1C_1)s \tag{154}$$

$$= G_1(C_4-K_0C_1)s$$

Table 5. The Relationships of Circuits of Figures 69 to 72

Figure	C_1	C_2	C_3	C_4	C_5
69	1	1	1	1	1
70	1	1	0	1	1
71	1	1	1	0	1
72	1	1	0	0	1

Table 6. The Relationships of Circuits of Figures 73 to 76

Figure	R_1	R_2	R_3	R_4	R_5
73	1	1	1	1	1
74	1	1	0	1	1
75	1	1	1	0	1
76	1	1	0	0	1

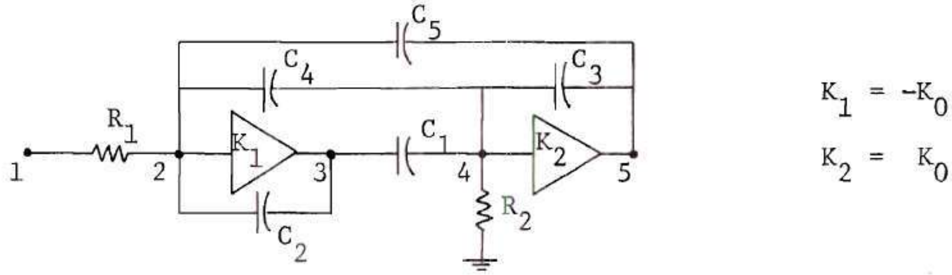


Figure 69. A Band-pass Filter Circuit

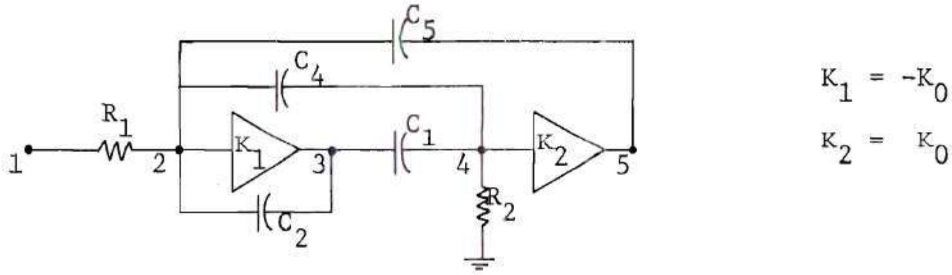


Figure 70. A Band-pass Filter Circuit

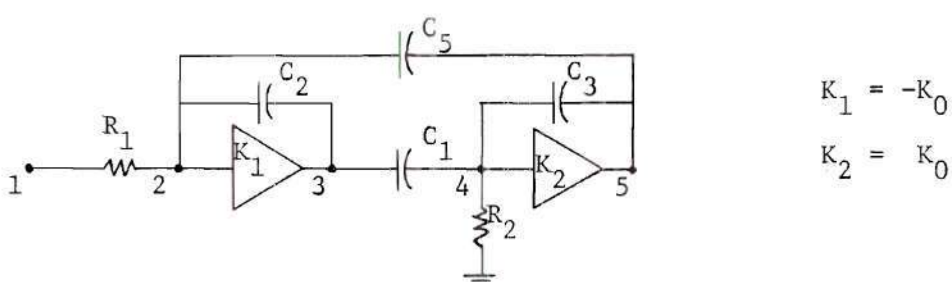


Figure 71. A Band-pass Filter Circuit

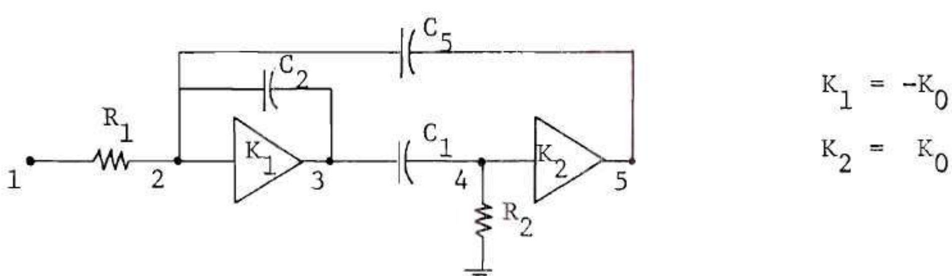
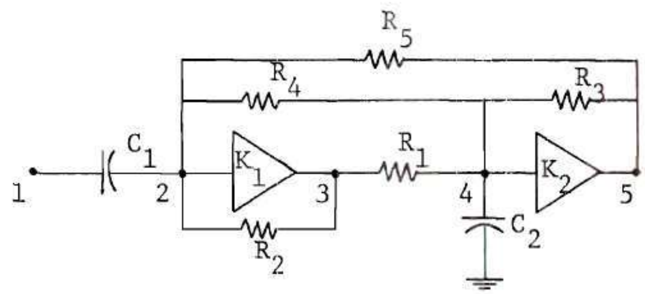


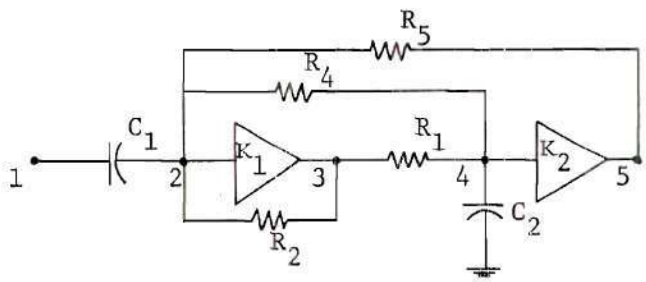
Figure 72. A Band-pass Filter Circuit



$K_1 = -K_0$

$K_2 = K_0$

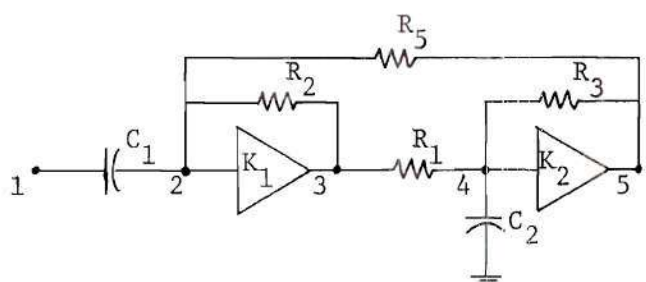
Figure 73. A Band-pass Filter Circuit



$K_1 = -K_0$

$K_2 = K_0$

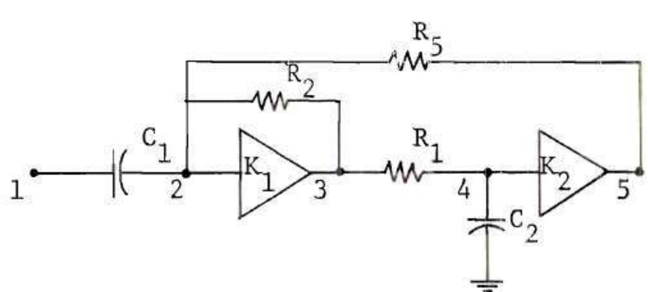
Figure 74. A Band-pass Filter Circuit



$K_1 = -K_0$

$K_2 = K_0$

Figure 75. A Band-pass Filter Circuit



$K_1 = -K_0$

$K_2 = K_0$

Figure 76. A Band-pass Filter Circuit

$$
\begin{aligned}
D(s) = {} & s^2\{C_2C_3(1+K_1K_2)+C_1C_5(1-K_1K_2) \\
& +(C_1C_2+C_2C_4+C_1C_4)(1-K_1) \\
& +(C_3C_4+C_3C_5+C_4C_5)(1-K_2)\} \\
& +s\{G_1(C_1+C_3+C_4)+G_2(C_2+C_4+C_5) \\
& -K_1G_2C_2-K_2G_1C_3\} + G_1G_2 \\
= {} & s^2\{C_2C_3(1-K_0^2)+C_1C_5(1+K_0^2) \\
& +(C_1C_2+C_2C_4+C_1C_4)(1+K_0) \\
& +(C_3C_4+C_3C_5+C_4C_5)(1-K_0)\} \\
& +s\{G_1(C_1+C_3+C_4)+G_2(C_2+C_4+C_5) \\
& +K_0G_2C_2-K_0G_1C_3\} + G_1G_2
\end{aligned}
\tag{155}
$$

For the circuits of Figures 70 to 72, the voltage-transfer functions can be found by using equations (154), (155), and Table 5. For example, N(s) and D(s) of Figure 70 are those of Figure 69 with C_3 set equal to zero.

For the circuit of Figure 73:

$$N(s) = C_1(G_4-K_0G_1)s \tag{156}$$

$$D(s) = s^2C_1C_2+s\{C_1(G_1+G_3+G_4)+C_2(G_2+G_4+G_5)+K_0C_2G_2 -K_0C_1G_3\}+G_2G_3(1-K_0^2)+G_1G_5(1+K_0^2)+(G_1G_2+G_2G_4 +G_1G_4)(1+K_0)+(G_3G_4+G_3G_5+G_4G_5)(1-K_0) \tag{157}$$

For the circuits of Figures 74 to 76, the voltage-transfer functions can be found by using equations (156), (157), and Table 6. For example, N(s) and D(s) of Figure 74 are those of Figure 73 with G_3 set equal to zero.

(iv) If we choose $K_1 = -K_0^2$ and $K_2 = \frac{1}{K_0}$, then from (128) and (131)

$$D(s) = (y_{22}y_{44}-y_{24}y_{42})-K_0(y_{23}y_{45}-y_{25}y_{43}) -K_0^2(y_{23}y_{44}-y_{24}y_{43})+\frac{1}{K_0}(y_{22}y_{45}-y_{25}y_{42}) \tag{158}$$

$$N(s) = y_{21}(y_{42}-K_0^2y_{43}) \tag{159}$$

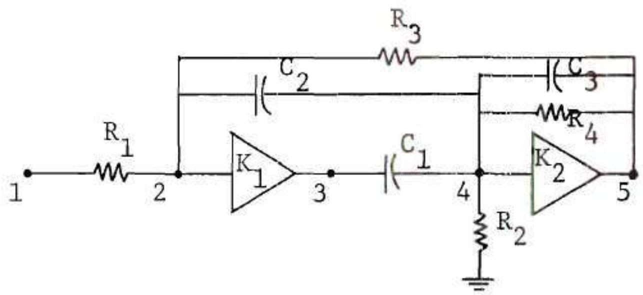
Equations (158) and (159) will be the basic equations for realizing the filters. After searching all possible combinations of y's, ten circuits as shown in Figures 77 to 86 are found. However, it is most convenient to present Figures 77 to 81 by Table 7 and their duals, Figures 82 to 86,

Table 7. The Relationships of Circuits of Figures 77 to 81

Figure	C_3	R_2	R_4
77	1	1	1
78	1	0	1
79	0	0	1
80	1	1	0
81	0	1	0

Table 8. The relationships of Circuits of Figures 82 to 86

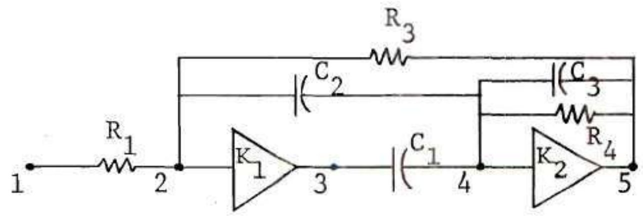
Figure	R_3	C_2	C_4
82	1	1	1
83	1	0	1
84	0	0	1
85	1	1	0
86	0	1	0



$K_1 = -K_0^2$

$K_2 = \frac{1}{K_0}$

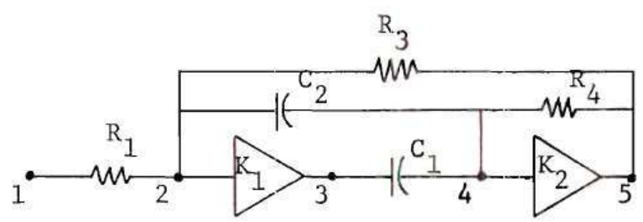
Figure 77. A Band-pass Filter Circuit



$K_1 = -K_0^2$

$K_2 = \frac{1}{K_0}$

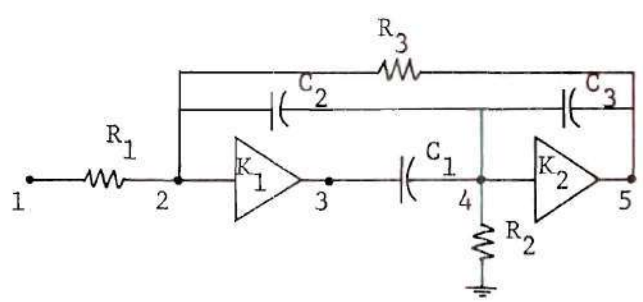
Figure 78. A Band-pass Filter Circuit



$K_1 = -K_0^2$

$K_2 = \frac{1}{K_0}$

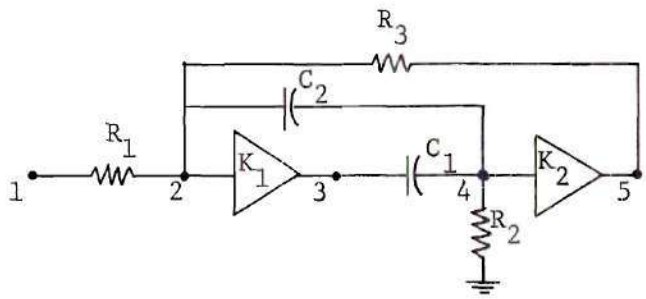
Figure 79. A Band-pass Filter Circuit



$K_1 = -K_0^2$

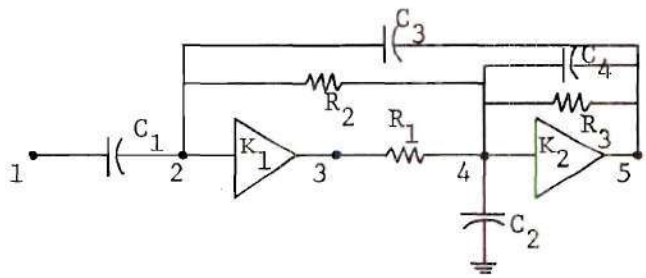
$K_2 = \frac{1}{K_0}$

Figure 80. A Band-pass Filter Circuit



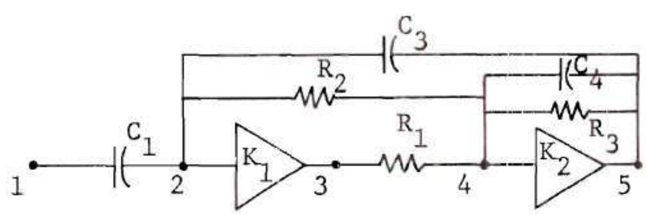
$$K_1 = -K_0^2 \qquad K_2 = \frac{1}{K_0}$$

Figure 81. A Band-pass Filter Circuit



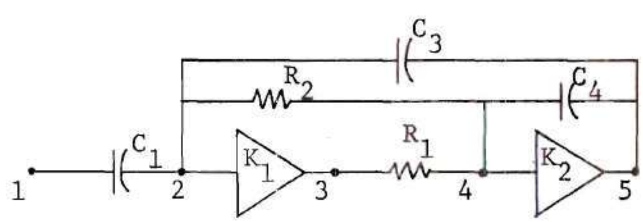
$$K_1 = -K_0^2 \qquad K_2 = \frac{1}{K_0}$$

Figure 82. A Band-pass Filter Circuit



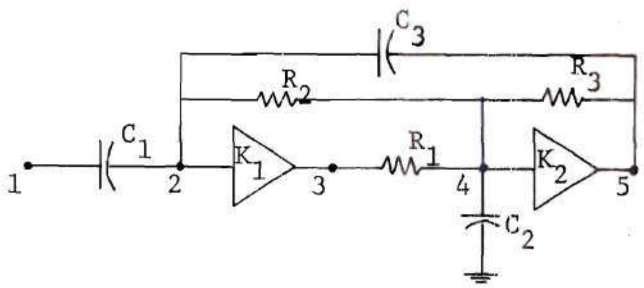
$$K_1 = -K_0^2 \qquad K_2 = \frac{1}{K_0}$$

Figure 83. A Band-pass Filter Circuit



$$K_1 = -K_0^2 \qquad K_2 = \frac{1}{K_0}$$

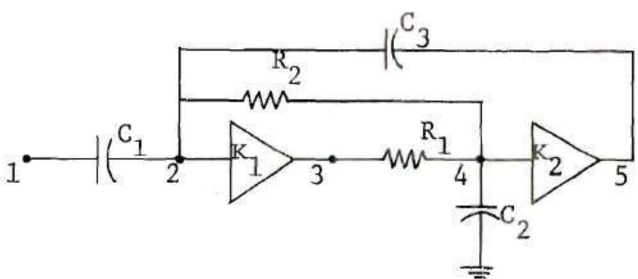
Figure 84. A Band-pass Filter Circuit



$$K_1 = -K_0^2$$

$$K_2 = \frac{1}{K_0}$$

Figure 85. A Band-pass Filter Circuit



$$K_1 = -K_0^2$$

$$K_2 = \frac{1}{K_0}$$

Figure 86. A Band-pass Filter Circuit

by Table 8. These tables show the basic circuits of each group and show which resistors or capacitors are retained in each figure.

For the circuit of Figure 77:

$$N(s) = G_1(C_2+K_1C_1)s \tag{160}$$

$$= G_1(C_2-K_0^2C_1)s$$

$$D(s) = s^2\{C_1C_2(1-K_1)+C_2C_3(1-K_2)\}+s\{C_2(G_2+G_4) \quad (161)$$

$$+(C_1+C_2+C_3)(G_1+G_3)-K_2C_3(G_1+G_3)-K_2C_2(G_3+G_4)$$

$$-K_1K_2G_3C_1\}+(G_1+G_3)(G_2+G_4)-K_2G_4(G_1+G_3)$$

$$= s\{C_1C_2(1+K_0^2)+C_2C_3(1-\frac{1}{K_0})\}+s\{C_2(G_2+G_4)$$

$$+(C_1+C_2+C_3)(G_1+G_3)-\frac{1}{K_0}\left(C_3(G_1+G_3)+C_2(G_3+G_4)\right)$$

$$+K_0G_3C_1\}+(G_1+G_3)\{G_2+G_4(1-\frac{1}{K_0})\}$$

For the circuits of Figures 78 to 80, the voltage-transfer functions can be found by using equations (160), (161), and Table 7, for example, N(s) and D(s) of Figure 78 are those of Figure 77 with G_2 set equal to zero.

For the circuit of Figure 82:

$$N(s) = C_1(G_2-K_0^2G_1)s \quad (162)$$

$$D(s) = s^2\{(C_1+C_3)\left(C_2+C_4(1-\frac{1}{K_0})\right)\}+s\{G_2(C_2+C_4)$$
$$+(G_1+G_2+G_3)(C_1+C_3)-\frac{1}{K_0}\left(G_3(C_1+C_3)+G_2(C_3+C_4)\right)$$
$$+K_0C_3G_1\}+G_1G_2(1+K_0^2)+G_2G_3(1-\frac{1}{K_0})$$

For the circuits of Figures 83 to 86, the voltage-transfer functions can be found by using equations (162), (163), and Table 8, for example, N(s) and D(s) of Figure 83 are those of Figure 82 with C_2 set equal to zero.

If the output is to be taken from node 2 instead of node 4, we can use the same procedures and obtain the same set of networks. This is equivalent to the interchanging of nodes 2 and 3 with nodes 4 and 5. Therefore, the band-pass sections for which the outputs are taken from node 2 will not be shown here.

By using the network configurations of Figures 5 and 6, and keeping the number of passive elements between each node and ground to be minimum, a total of 80 circuits are found possible for realizing the constant-Q, tunable band-pass sections. These circuits came from the 9 basic circuits, Figures 7, 8, 31, 43, 55, 56, 61, 69, and 77, and their dual circuits, Figures 11, 12, 33, 49, 58, 59, 65, 73, and 82.

Since the number of the basic circuits is not large, all circuits were found without the use of computer programming.

CHAPTER IV

DERIVATION OF LOW-PASS AND HIGH-PASS FILTER CIRCUITS

In this chapter we shall make a detailed study of second-order low-pass and high-pass filter sections based on the method developed in Chapter II. The general procedures will be similar to those of deriving band-pass filter circuits in Chapter III.

We shall synthesize these filter sections from the basic network configurations of Figures 3 and 4 with $n = 5$.

The low-pass and the high-pass filters will be derived concurrently since one is the dual of the order.

Low-pass and High-pass Filters Based on the Configuration of Figure 5

From the network configuration of Figure 5, the constant-Q, tunable active low-pass and high-pass filters can be obtained by making either terminal 3 (or equivalently, 4 or 5) as the output; or terminal 2 as the output.

Filters Obtained by Using Terminal 3 as Output

If the output is to be taken from terminal 3, the denominator and the numerator of the voltage-transfer function are shown in (104) and (105).

For this configuration to be either low-pass or high-pass sections, it is necessary that $N(s)$ be the function of s^0 only for low-pass section or $N(s)$ be the function of s^2 only for high-pass section. Therefore, we must have

$$y_{31} = 0 \tag{164}$$

Thus
$$N(s) = y_{21}y_{32} \tag{165}$$

(i) If we choose $K_1 = -K_0$ and $K_2 = K_0$, then D(s) will be as in (108). Equations (108) and (165) will be the basic equations for realizing the filters. After searching all possible combinations* of y's, two circuits as shown in Figures 87 and 88 are found possible for the low-pass sections.

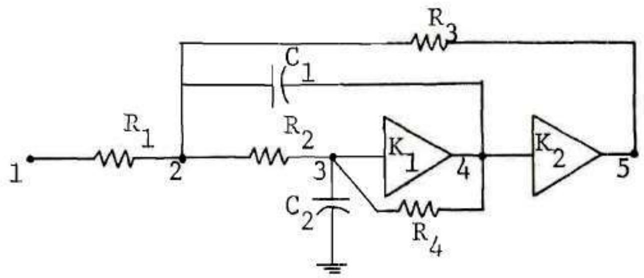
For the circuit of Figure 87:

$$N(s) = G_1G_2 \tag{166}$$

$$\begin{aligned} D(s) &= s^2C_1C_2+s\{C_1(G_2+G_4)+C_2(G_1+G_2+G_3)-K_1(C_1G_2+C_1G_4)\} \\ &\quad +G_1G_2+(G_1+G_2+G_3)G_4(1-K_1)+G_2G_3(1-K_1K_2) \\ &= s^2C_1C_2+s\{C_1(G_2+G_4)+C_2(G_1+G_2+G_3)+K_0C_1(G_2+G_4)\} \\ &\quad +G_1G_2+(G_1+G_2+G_3)G_4(1+K_0)+G_2G_3(1+K_0^2) \end{aligned} \tag{167}$$

---

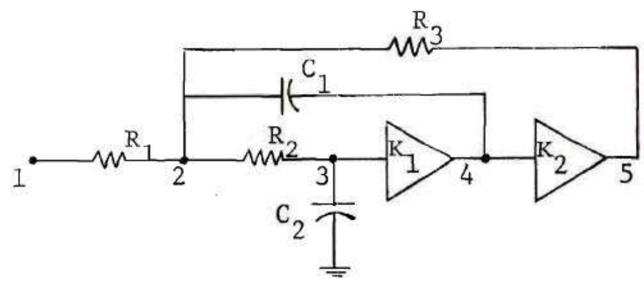
*The number of admittances between each node and ground is kept at the minimum. If we remove this restriction, the number of possible circuits can be increased indefinitely. For example, any admittance may be connected between the output of the VCVS and ground without altering the performance of the filter.



$K_1 = -K_0$

$K_2 = K_0$

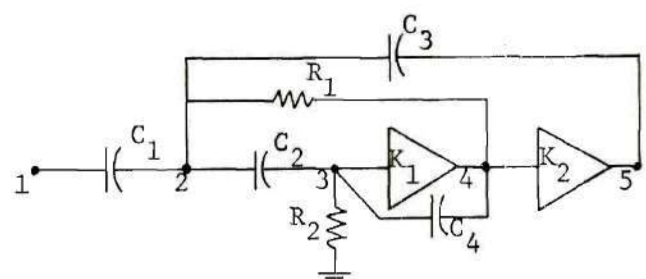
Figure 87. A Low-pass Filter Circuit



$K_1 = -K_0$

$K_2 = K_0$

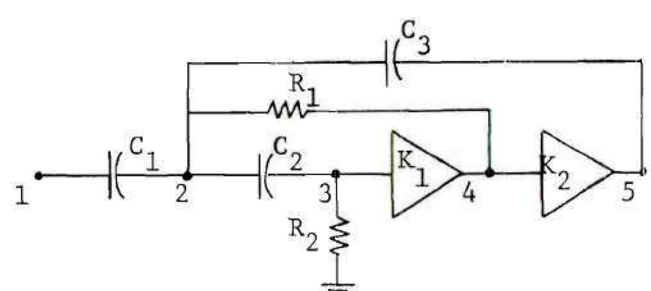
Figure 88. A Low-pass Filter Circuit



$K_1 = -K_0$

$K_2 = K_0$

Figure 89. A High-pass Filter Circuit



$K_1 = -K_0$

$K_2 = K_0$

Figure 90. A High-pass Filter Circuit

For the circuit of Figure 88, N(s) and D(s) will be the same as (166) and (167) with G_4 set equal to zero.

The duals of Figures 87 and 88 have the characteristics of the high-pass sections. They are shown in Figures 89 and 90.

For the circuit of Figure 89:

$$N(s) = s^2 C_1 C_2 \tag{168}$$

$$D(s) = s^2\{C_1C_2+(C_1+C_2+C_3)C_4(1+K_0)+C_2C_3(1+K_0^2)\} \tag{169}$$

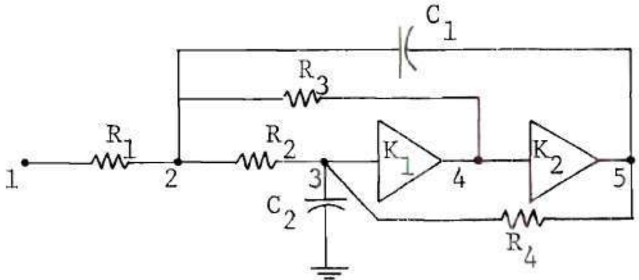
$$+s\{G_1(C_2+C_4)+G_2(C_1+C_2+C_3)+K_0G_1(C_2+C_4)\}+G_1G_2$$

For the circuit of Figure 90, N(s) and D(s) will be the same as (168) and (169) with C_4 set equal to zero.

(ii) If we choose $K_1 = -K_0^2$ and $K_2 = \frac{1}{K_0}$, then D(s) will be as in (117). Equations (117) and (165) will be the basic equations for realizing the filters. After searching all possible combinations of y's, two circuits as shown in Figures 91 and 92 are found possible for the low-pass sections. Their duals, Figures 93 and 94, have the characteristics of the high-pass sections.

Figures 91 to 94 are similar to Figures 87 to 90, with the interchanging of passive elements connected to nodes 4 and 5. The voltage-transfer functions of Figures 91 to 94, after the substitution of values of K_1 and K_2, are identical to those of Figures 87 to 90, respectively.

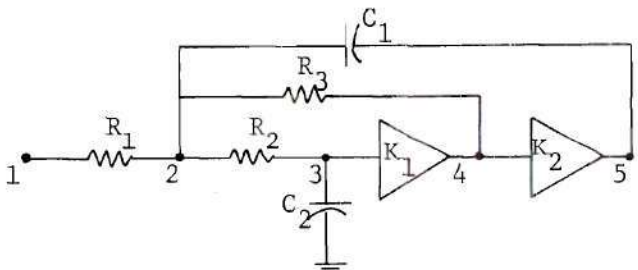
(iii) If the network configuration is modified such that $V_4 = K_1V_3$ and $V_5 = K_2V_3$, then another choice of K's would be $K_1 = -K_0$ and $K_2 = -K_0^2$.



$$K_1 = -K_0^2$$

$$K_2 = \frac{1}{K_0}$$

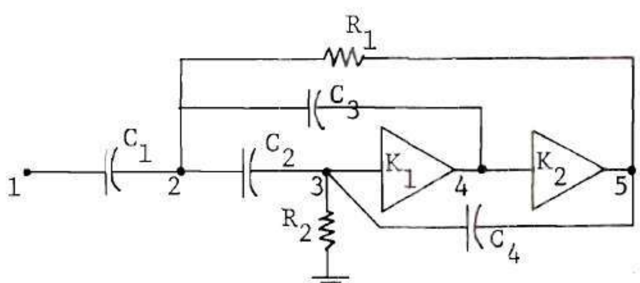
Figure 91. A Low-pass Filter Circuit



$$K_1 = -K_0^2$$

$$K_2 = \frac{1}{K_0}$$

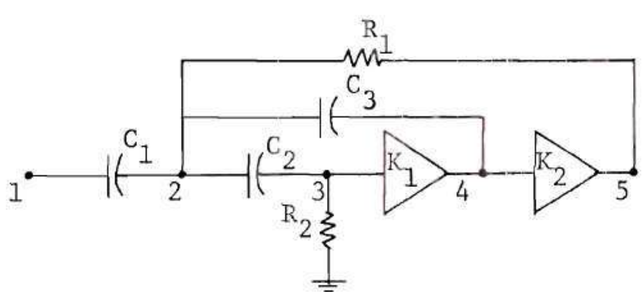
Figure 92. A Low-pass Filter Circuit



$$K_1 = -K_0^2$$

$$K_2 = \frac{1}{K_0}$$

Figure 93. A High-pass Filter Circuit



$$K_1 = -K_0^2$$

$$K_2 = \frac{1}{K_0}$$

Figure 94. A High-pass Filter Circuit

$D(s)$ will be as in (118). Equations (118) and (165) will be the basic equations for realizing the filters. After searching all possible combinations of y's, two circuits as shown in Figure 95 and 96 are found possible for the low-pass sections. Their duals, Figures 97 and 98, have the characteristics of the high-pass sections.

Figures 95 to 98 are similar to Figures 87 to 90 with changes in locations of K_2. The voltage-transfer functions of Figures 95 to 98, after the substitutions of values of K_1 and K_2, are identical to those of Figures 87 to 90, respectively.

Filters Obtained by Using Terminal 2 as Output

If the output is to be taken from terminal 2, the denominator and the numerator of the voltage-transfer function are shown in (104) and (119).

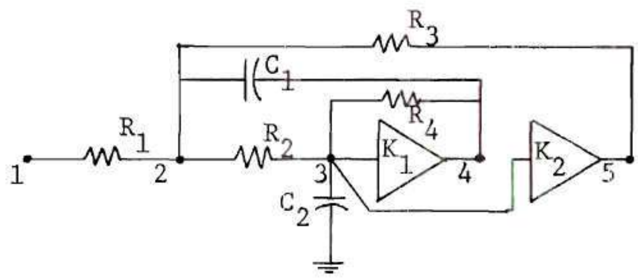
For this configuration to be either low-pass or high-pass sections, it is necessary that $N(s)$ be the function of s^0 only for low-pass section or $N(s)$ be the function of s^2 only for high-pass section. Therefore, we must have

$$y_{21} = 0 \tag{170}$$

Thus

$$N(s) = y_{31}(y_{23}+K_1y_{24}+K_1K_2y_{25}) \tag{171}$$

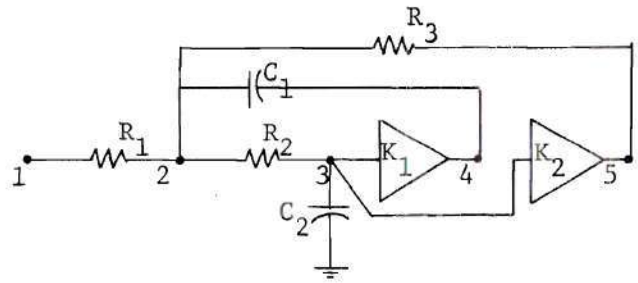
(i) If we choose $K_1 = -K_0$ and $K_2 = K_0$, then $D(s)$ will be as in (108). Equations (108) and (171) will be the basic equations for realizing the filters. After searching all possible combinations of y's, two circuits as shown in Figures 99 and 100 are found possible for the low-pass sections.



$K_1 = -K_0$

$K_2 = -K_0^2$

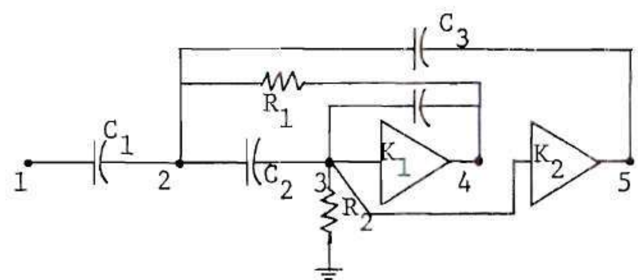
Figure 95. A Low-pass Filter Circuit



$K_1 = -K_0$

$K_2 = -K_0^2$

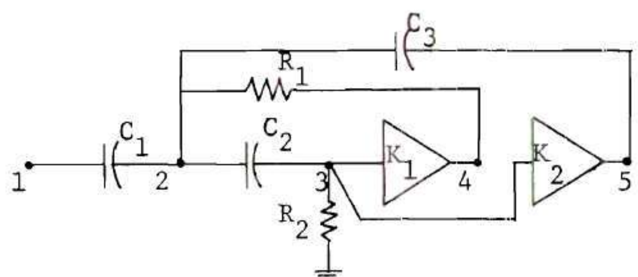
Figure 96. A Low-pass Filter Circuit



$K_1 = -K_0$

$K_2 = -K_0^2$

Figure 97. A High-pass Filter Circuit



$K_1 = -K_0$

$K_2 = -K_0^2$

Figure 98. A High-pass Filter Circuit

For the circuit of Figure 99:

$$N(s) = G_1(G_2+K_1G_3+K_1K_2G_4) \tag{172}$$

$$= G_1(G_2-K_0G_3-K_0^2G_4)$$

$$D(s) = s^2C_1C_2+s\{C_2(G_1+G_2+G_5)+C_1(G_2+G_3+G_4)-C_2G_5K_1\} \tag{173}$$

$$+(G_2+G_3+G_4)\{G_1+G_5(1-K_1)\}+G_2G_3(1-K_1)+G_2G_4(1-K_1K_2)$$

$$= s^2C_1C_2+s\{C_2(G_1+G_2+G_5)+C_1(G_2+G_3+G_4)+C_2G_5K_0\}$$

$$+(G_2+G_3+G_4)\{G_1+G_5(1+K_0)\}+G_2G_3(1+K_0)+G_2G_4(1+K_0^2)$$

For the circuit of Figure 100, N(s) and D(s) will be the same as (172) and (173) with G_3 set equal to zero.

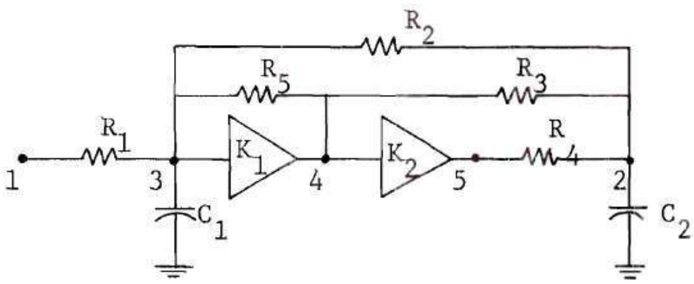
The duals of Figures 99 and 100 have the characteristics of the high-pass sections. They are shown in Figures 101 and 102.

For the circuit of Figure 101:

$$N(s) = C_1(C_2-K_0C_3-K_0^2C_4)s^2 \tag{174}$$

$$D(s) = s^2\{(C_2+C_3+C_4)\{C_1+C_5(1+K_0)\}+C_2C_3(1+K_0)+C_2C_4(1+K_0^2)\} \tag{175}$$

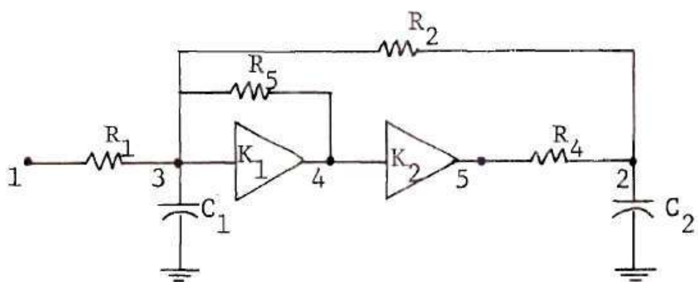
$$+s\{G_2(C_1+C_2+C_5)+G_1(C_2+C_3+C_4)+G_2C_5K_0\}+G_1G_2$$



$K_1 = -K_0$

$K_2 = K_0$

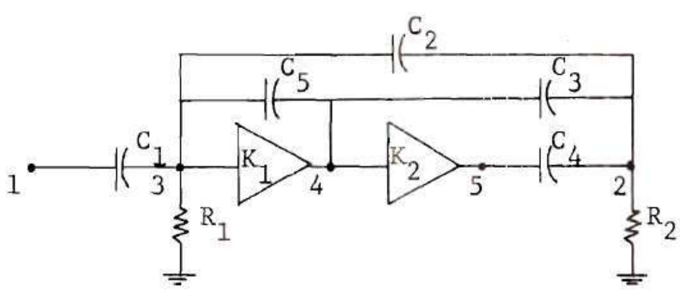
Figure 99. A Low-pass Filter Circuit



$K_1 = -K_0$

$K_2 = K_0$

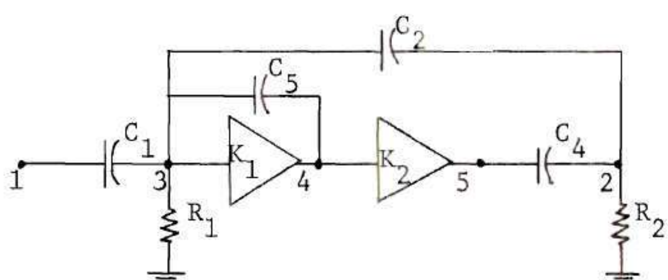
Figure 100. A Low-pass Filter Circuit



$K_1 = -K_0$

$K_2 = K_0$

Figure 101. A High-pass Filter Circuit



$K_1 = -K_0$

$K_2 = K_0$

Figure 102. A High-pass Filter Circuit

For the circuit of Figure 102, N(s) and D(s) will be the same as (174) and (175) with C_3 set equal to zero.

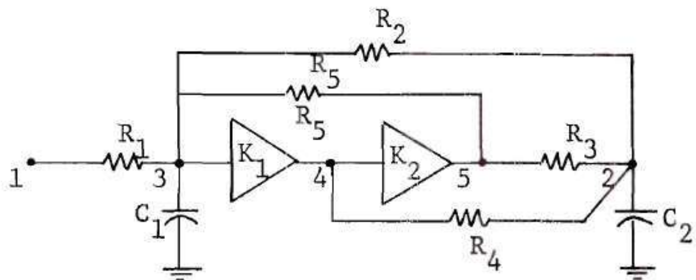
(ii) If we choose $K_1 = -K_0^2$ and $K_2 = \frac{1}{K_0}$, then D(s) will be as in (117). Equations (117) and (171) will be the basic equations for realizing the filters. After searching all possible combinations of y's, two circuits as shown in Figures 103 and 104 are found possible for the low-pass sections. Their duals, Figures 105 and 106, have the characteristics of the high-pass sections.

Figures 103 to 106 are similar to Figures 99 to 102 with the interchanging of passive elements connected to nodes 4 and 5. The voltage-transfer functions of Figures 103 and 106, after the substitution of values of K_1 and K_2, are identical to those of Figures 99 to 102, respectively.

(iii) If the network configuration is modified such that $V_4 = K_1V_3$ and $V_5 = K_2V_3$, then another choice of K's would be $K_1 = -K_0$ and $K_2 = -K_0^2$. D(s) will be as in (118). Equations (118) and (171) will be the basic equations for realizing the filters. After searching all possible combinations of y's, two circuits as shown in Figures 107 and 108 are found possible for the low-pass sections. Their duals, Figures 109 and 110, have the characteristics of the high-pass sections.

Figures 107 to 110 are similar to Figures 99 to 102 with changes in locations of K_2. The voltage-transfer functions of Figures 107 to 110, after the substitution of values of K_1 and K_2, are identical to those of Figures 99 to 102, respectively.

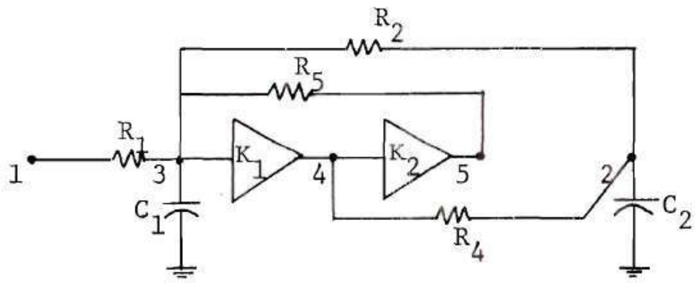
Low-pass and High-pass Filters Based on the Configuration of Figure 6



$K_1 = -K_0^2$

$K_2 = \frac{1}{K_0}$

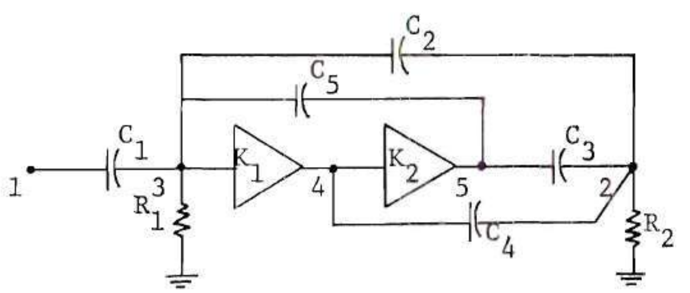
Figure 103. A Low-pass Filter Circuit



$K_1 = -K_0^2$

$K_2 = \frac{1}{K_0}$

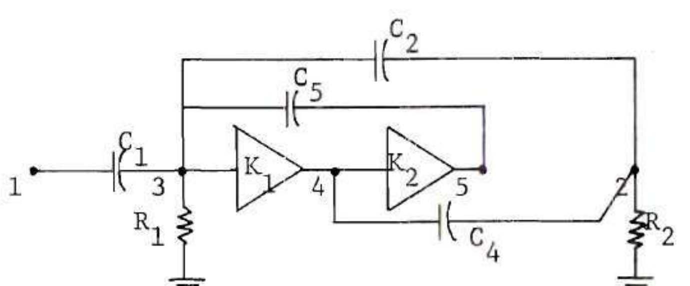
Figure 104. A Low-pass Filter Circuit



$K_1 = -K_0^2$

$K_2 = \frac{1}{K_0}$

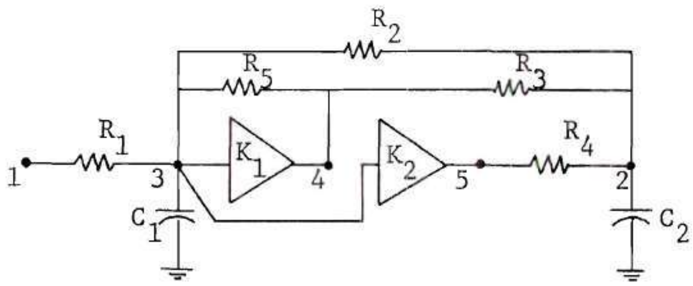
Figure 105. A High-pass Filter Circuit



$K_1 = -K_0^2$

$K_2 = \frac{1}{K_0}$

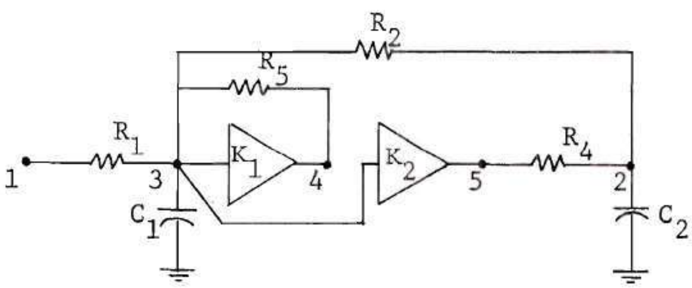
Figure 106. A High-pass Filter Circuit



$K_1 = -K_0$

$K_2 = -K_0^2$

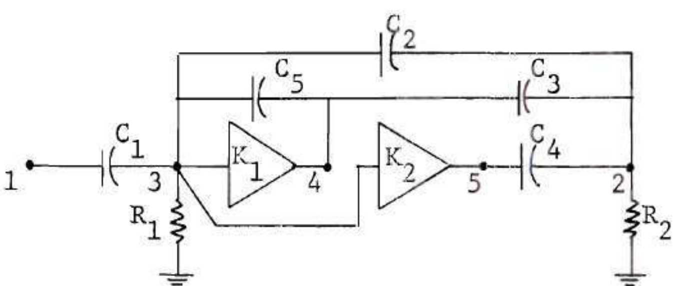
Figure 107. A Low-pass Filter Circuit



$K_1 = -K_0$

$K_2 = -K_0^2$

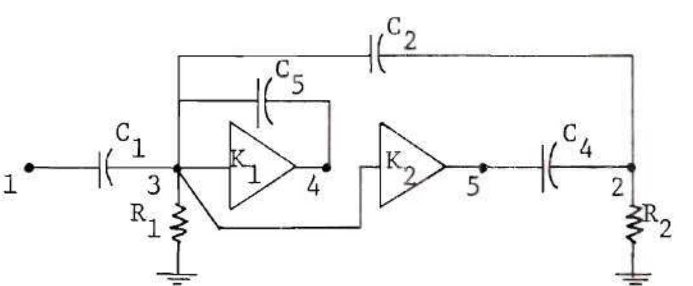
Figure 108. A Low-pass Filter Circuit



$K_1 = -K_0$

$K_2 = -K_0^2$

Figure 109. A High-pass Filter Circuit



$K_1 = -K_0$

$K_2 = -K_0^2$

Figure 110. A High-pass Filter Circuit

If the network configuration of Figure 6 is used, the output may be taken from terminals 2, 3, 4, or 5. However, it should become apparent that these options are all basically equivalent to one another.

If the output is to be taken from node 4, then the denominator and the numerator of the voltage-transfer function will be as in equations (128) and (129).

For this configuration to be either low-pass or high-pass sections, it is necessary that

$$y_{41} = 0 \tag{176}$$

Thus
$$N(s) = y_{21}(y_{42}+K_1 y_{43}) \tag{177}$$

(i) If we choose $K_1 = K_2 = -K_0$ and substitute into (128), then this equation and (177) will be the basic equations for realizing the filters. After searching all possible combinations of y's, seven circuits as shown in Figures 111 to 117 are found possible for the low-pass sections. Their duals, Figures 118 to 124, have the characteristics of the high-pass sections.

However, it is most convenient to present Figures 113 to 117 and Figures 120 to 124 by Tables 9 and 10, respectively. These tables show the basic circuits of each group, and show which resistors or capacitors are retained in each figure, for example, Figure 114 is derived from Figure 113 with R_2 and R_4 deleted. The voltage-transfer function of Figure 114 is that of Figure 113 with G_2 and G_4 set equal to zero.

For the circuit of Figure 111:

$$N(s) = G_1G_2K_1 \tag{178}$$

$$= -G_1G_2K_0$$

$$D(s) = s^2C_1C_2(1-K_1-K_2+K_1K_2)+s(G_1C_2 \tag{179}$$

$$+G_2C_1-K_1G_2C_1-K_2G_1C_2)+G_1G_2$$

$$= s^2C_1C_2(1+2K_0+K_0^2)+s(G_1C_2$$

$$+G_2C_1)(1+K_0)+G_1G_2$$

For the circuit of Figure 112:

$$N(s) = G_1G_2 \tag{180}$$

$$D(s) = s^2C_1C_2(1-K_1-K_2+K_1K_2)+s\{C_1G_2(1-K_1) \tag{181}$$

$$+C_2(G_1+G_2)(1-K_2)\}+G_1G_2$$

$$= s^2C_1C_2(1+2K_0+K_0^2)+s\{C_1G_2+C_2(G_1$$

$$+G_2)\}(1+K_0)+G_1G_2$$

For the circuit of Figure 113:

$$N(s) = G_1(G_2+K_1G_3) \tag{182}$$

$$= G_1(G_2-K_0G_3)$$

$$D(s) = s^2C_1C_2+s\{C_1(G_2+G_3+G_6)+C_2(G_1+G_2+G_4+G_5)-K_1C_2G_5 \tag{183}$$

$$-K_2C_1G_6\}+G_1G_2+G_1G_3+G_2G_4+(1-K_1)(G_2G_3+G_2G_5+G_3G_5)$$

$$+(1-K_2)(G_1G_6+G_2G_6+G_4G_6)+G_3G_4(1-K_1K_2)+G_5G_6(1-K_1$$

$$-K_2+K_1K_2)$$

$$= s^2C_1C_2+s\{C_1(G_2+G_3+G_6)+C_2(G_1+G_2+G_4+G_5)+K_0(C_2G_5$$

$$+C_1G_6)\}+G_1G_2+G_1G_3+G_2G_4+(1+K_0)(G_2G_3+G_2G_5+G_3G_5$$

$$+G_1G_6+G_2G_6+G_4G_6)+G_3G_4(1-K_0^2)+G_5G_6(1+2K_0+K_0^2)$$

For the circuits of Figures 114 to 117, N(s) and D(s) can be found by using equations (182), (183), and Table 9.

For the circuit of Figure 118:

$$N(s) = -C_1C_2K_0s^2 \tag{184}$$

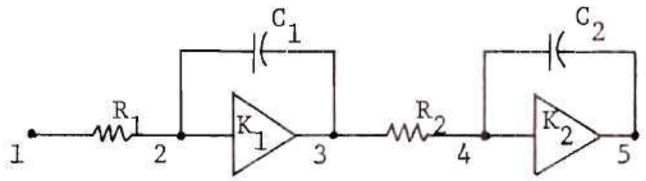
$$D(s) = s^2C_1C_2+s(G_2C_1+G_1C_2)(1+K_0)+G_1G_2(1+2K_0+K_0^2) \tag{185}$$

Table 9. The Relationships of Circuits of Figures 113 to 117

Figure	R_1	R_2	R_3	R_4	R_5	R_6
113	1	1	1	1	1	1
114	1	0	1	0	1	1
115	1	0	1	1	1	1
116	1	1	0	0	1	1
117	1	1	0	1	1	1

Table 10. The Relationships of Circuits of Figures 120 to 124

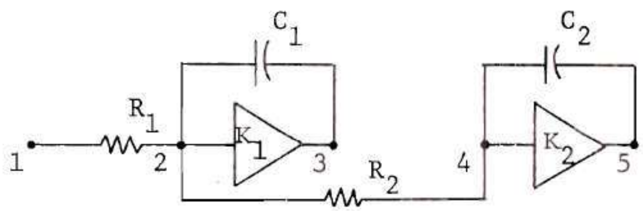
Figure	C_1	C_2	C_3	C_4	C_5	C_6
120	1	1	1	1	1	1
121	1	0	1	0	1	1
122	1	0	1	1	1	1
123	1	1	0	0	1	1
124	1	1	0	1	1	1



$K_1 = -K_0$

$K_2 = -K_0$

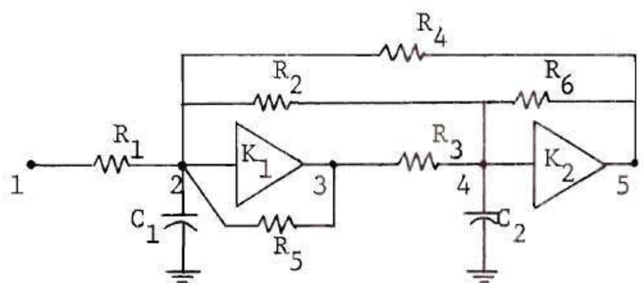
Figure 111. A Low-pass Filter Circuit



$K_1 = -K_0$

$K_2 = -K_0$

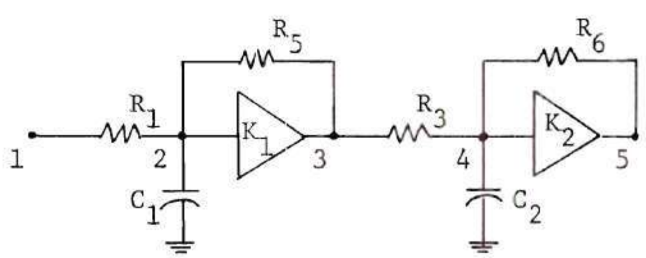
Figure 112. A Low-pass Filter Circuit



$K_1 = -K_0$

$K_2 = -K_0$

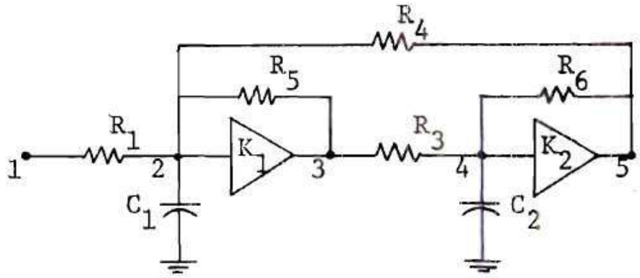
Figure 113. A Low-pass Filter Circuit



$K_1 = -K_0$

$K_2 = -K_0$

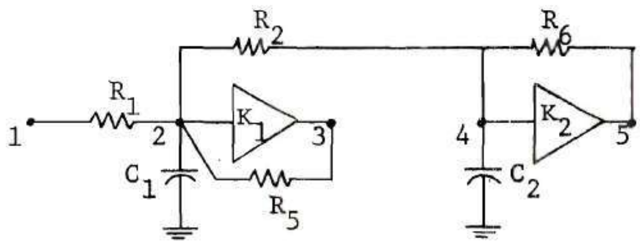
Figure 114. A Low-pass Filter Circuit



$K_1 = -K_0$

$K_2 = -K_0$

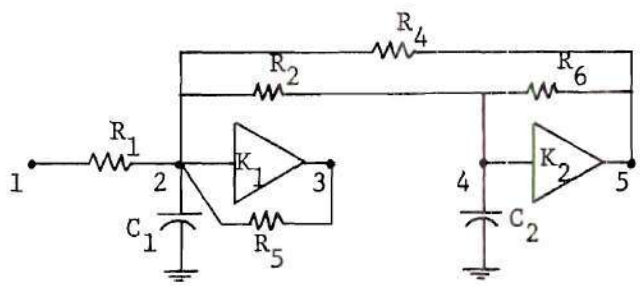
Figure 115. A Low-pass Filter Circuit



$K_1 = -K_0$

$K_2 = -K_0$

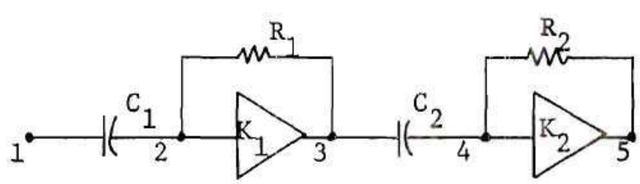
Figure 116. A Low-pass Filter Circuit



$K_1 = -K_0$

$K_2 = -K_0$

Figure 117. A Low-pass Filter Circuit



$K_1 = -K_0$

$K_2 = -K_0$

Figure 118. A High-pass Filter Circuit

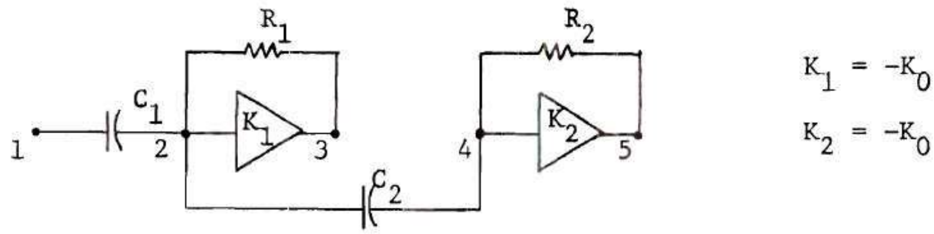


Figure 119. A High-pass Filter Circuit

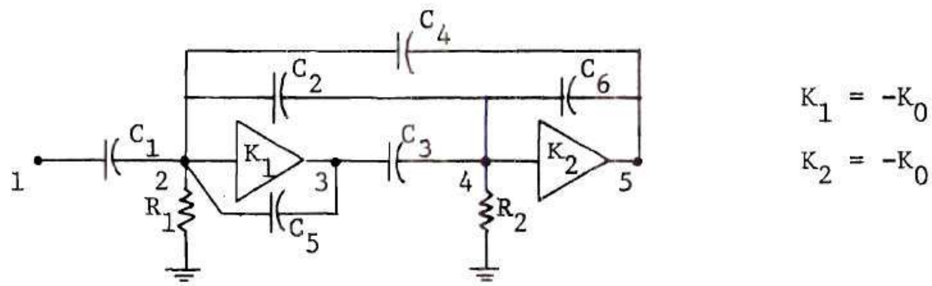


Figure 120. A High-pass Filter Circuit

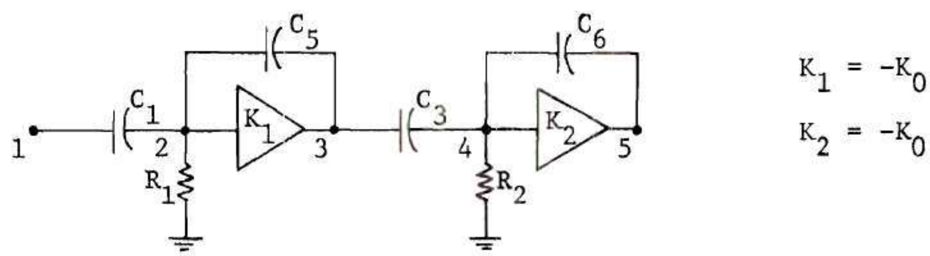


Figure 121. A High-pass Filter Circuit

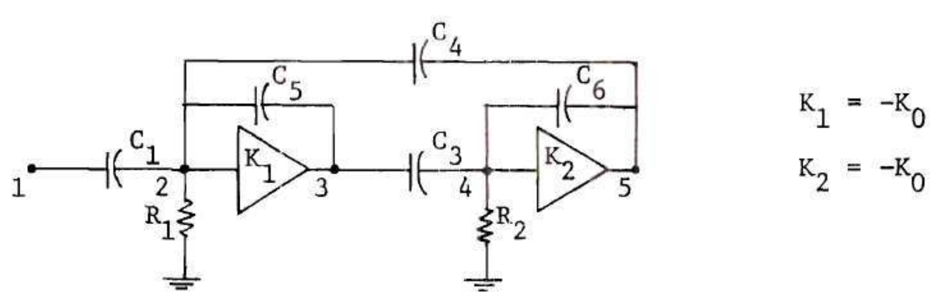
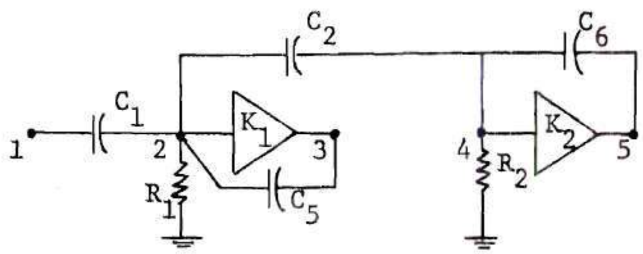


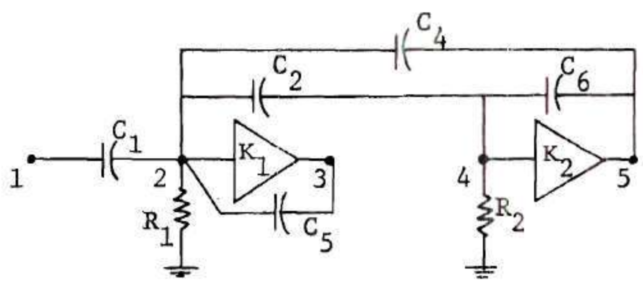
Figure 122. A High-pass Filter Circuit



$K_1 = -K_0$

$K_2 = -K_0$

Figure 123. A High-pass Filter Circuit



$K_1 = -K_0$

$K_2 = -K_0$

Figure 124. A High-pass Filter Circuit

For the circuit of Figure 119:

$$N(s) = C_1C_2s^2 \tag{186}$$

$$D(s) = s^2C_1C_2+s\{G_1C_2+G_2(C_1+C_2)\} \tag{187}$$

$$(1+K_0)+G_1G_2(1+2K_0+K_0^2)$$

For the circuit of Figure 120:

$$N(s) = C_1(C_2-K_0C_3)s^2 \tag{188}$$

$$D(s) = s^2\{C_1C_2+C_1C_3+C_2C_4+(1+K_0)(C_2C_3+C_2C_5+C_3C_5+C_1C_6 \quad (189)$$

$$+C_2C_6+C_4C_6)+C_3C_4(1-K_0^2)+C_5C_6(1+2K_0+K_0^2)\}$$

$$+s\{G_1(C_2+C_3+C_6)+G_2(C_1+C_2+C_4+C_5)+K_0(G_2C_5+G_1C_6)\}$$

$$+G_1G_2$$

For the circuits of Figures 121 to 124, N(s) and D(s) can be found by using equations (188), (189), and Table 10.

(ii) If we choose $K_1 = K_0$, $K_2 = -K_0$ and substitute into (128), then this equation and (177) will be the basic equations for realizing the filters. After searching all possible combinations of y's, four circuits as shown in Figures 125 to 128 are found possible for the low-pass sections. Their duals, Figures 129 to 132, have the characteristics of the high-pass sections.

However, it is most convenient to present Figures 125 to 128 and Figures 129 to 132 by Table 11 and 12, which show the basic circuit for each group and show which resistors or capacitors are retained in each figure.

Figure 125 has the same passive structure as that of Figure 113. The voltage-transfer function of Figure 125 can be found by substitute $K_1 = K_0$ and $K_2 = -K_0$ into the general equations of Figure 113.

For the circuit of Figure 125:

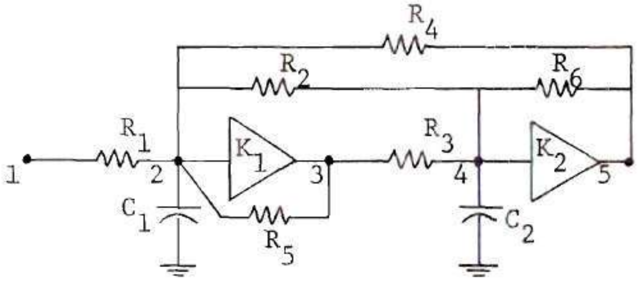
$$N(s) = G_1(G_2+K_0G_3) \quad (190)$$

Table 11. The relationships of Circuits of Figures 125 to 128

Figure	R_1	R_2	R_3	R_4	R_5	R_6
125	1	1	1	1	1	1
126	1	1	1	1	0	1
127	1	0	1	1	1	1
128	1	0	1	1	0	1

Table 12. The Relationships of Circuits of Figures 129 to 132

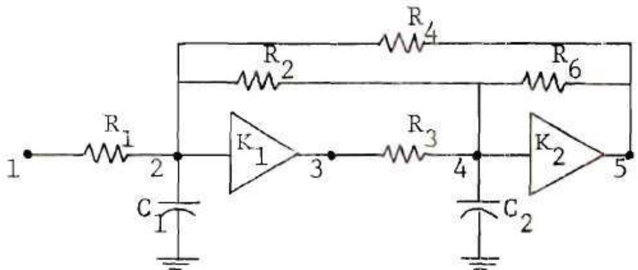
Figure	C_1	C_2	C_3	C_4	C_5	C_6
129	1	1	1	1	1	1
130	1	1	1	1	0	1
131	1	0	1	1	1	1
132	1	0	1	1	0	1



$K_1 = K_0$

$K_2 = -K_0$

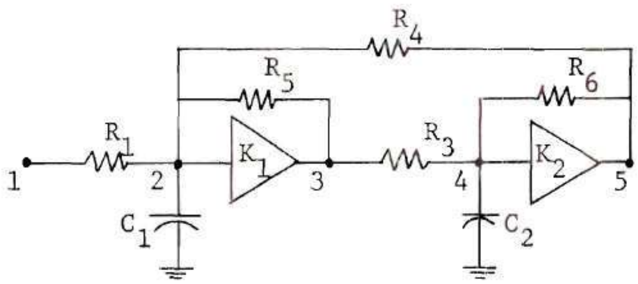
Figure 125. A Low-pass Filter Circuit



$K_1 = K_0$

$K_2 = -K_0$

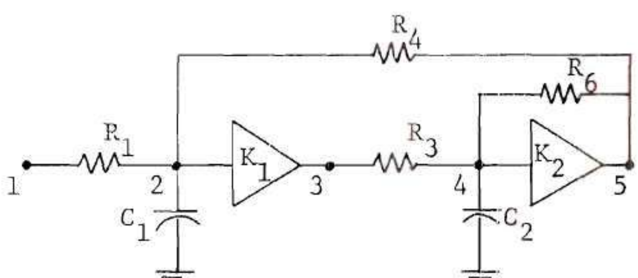
Figure 126. A Low-pass Filter Circuit



$K_1 = K_0$

$K_2 = -K_0$

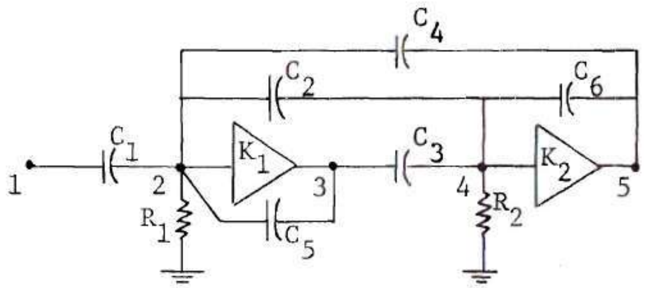
Figure 127. A Low-pass Filter Circuit



$K_1 = K_0$

$K_2 = -K_0$

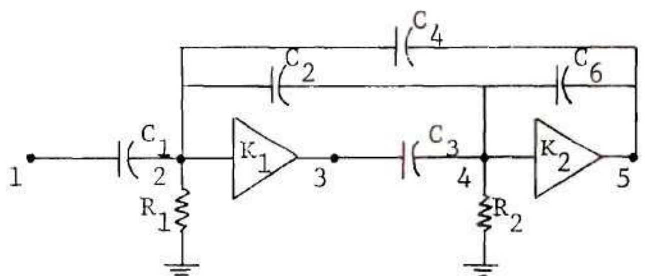
Figure 128. A Low-pass Filter Circuit



$K_1 = K_0$

$K_2 = -K_0$

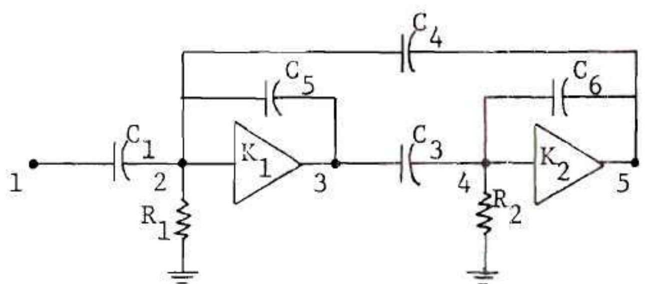
Figure 129. A High-pass Filter Circuit



$K_1 = K_0$

$K_2 = -K_0$

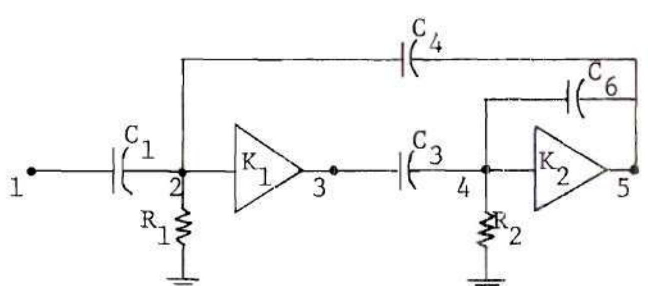
Figure 130. A High-pass Filter Circuit



$K_1 = K_0$

$K_2 = -K_0$

Figure 131. A High-pass Filter Circuit



$K_1 = K_0$

$K_2 = -K_0$

Figure 132. A High-pass Filter Circuit

$$D(s) = s^2C_1C_2+s\{C_1(G_2+G_3+G_6)+C_2(G_1+G_2+G_4+G_5)-K_0C_2G_5 \tag{191}$$

$$+K_0C_1G_6\}+G_1G_2+G_1G_3+G_2G_4+(1-K_0)(G_2G_3+G_2G_5+G_3G_5)$$

$$+(1+K_0)(G_1G_6+G_2G_6+G_4G_6)+G_3G_4(1+K_0^2)+G_5G_6(1-K_0^2)$$

For the circuits of Figures 126 to 128, $N(s)$ and $D(s)$ can be found using equations (190), (191), and Table 11.

For the circuit of Figure 129:

$$N(s) = C_1(C_2+K_0C_3)s^2 \tag{192}$$

$$D(s) = s^2\{C_1C_2+C_1C_3+C_2C_4+(1-K_0)(C_2C_3+C_2C_5+C_3C_5) \tag{193}$$

$$+(1+K_0)(C_1C_6+C_2C_6+C_4C_6)+C_3C_4(1+K_0^2)+C_5C_6(1-K_0^2)\}$$

$$+s\{G_1(C_2+C_3+C_6)+G_2(C_1+C_2+C_4+C_5)-K_0G_2C_5+K_0G_1C_6\}+G_1G_2$$

For the circuits of Figures 130 to 132, $N(s)$ and $D(s)$ can be found using equations (192), (193), and Table 12.

(iii) If we choose $K_1 = -K_0$, $K_2 = K_0$ and substitute into (128), then this equation and (177) will be the basic equations for realizing the filters. After searching all possible combinations of y's, four circuits as shown in Figures 133 to 136 are found possible for the low-pass sections. Their duals, Figures 137 to 140, have the characteristics of the high-pass sections.

It is most convenient to present Figures 133 to 136 and Figures 137 to 140 by Tables 13 and 14, which show the basic circuit for each group and show which resistors or capacitors are retained in each figure.

Figure 133 has the same passive structure as that of Figure 113. The voltage-transfer function of Figure 133 can be found by substitute $K_1 = -K_0$ and $K_2 = K_0$ into the general equations of Figure 113.

For the circuit of Figure 133:

$$N(s) = G_1(G_2-K_0G_3) \tag{194}$$

$$\begin{aligned} D(s) = {} & s^2C_1C_2+s\{C_1(G_2+G_3+G_6)+C_2(G_1+G_2+G_4+G_5)+K_0C_2G_5 \\ & -K_0C_1G_6\}+G_1G_2+G_1G_3+G_2G_4+(1+K_0)(G_2G_3+G_2G_5+G_3G_5) \\ & +(1-K_0)(G_1G_6+G_2G_6+G_4G_6)+G_3G_4(1+K_0^2)+G_5G_6(1-K_0^2) \end{aligned} \tag{195}$$

For the circuits of Figures 134 to 136, N(s) and D(s) can be found using equations (194), (195), and Table 13.

For the circuit of Figure 137:

$$N(s) = C_1(C_2-K_0C_3)s^2 \tag{196}$$

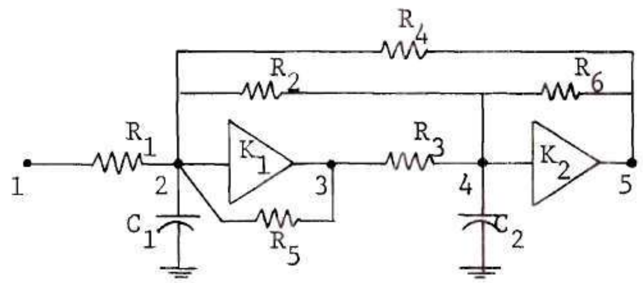
$$\begin{aligned} D(s) = {} & s^2\{C_1C_2+C_1C_3+C_2C_4+(1+K_0)(C_2C_3+C_2C_5+C_3C_5) \\ & +(1-K_0)(C_1C_6+C_2C_6+C_4C_6)+C_3C_4(1+K_0^2)+C_5C_6(1-K_0^2)\} \end{aligned} \tag{197}$$

Table 13. The Relationships of Circuits of Figures 133 to 136

Figure	R_1	R_2	R_3	R_4	R_5	R_6
133	1	1	1	1	1	1
134	1	1	1	1	1	0
135	1	0	1	1	1	1
136	1	0	1	1	1	0

Table 14. The Relationships of Circuits of Figures 137 to 140

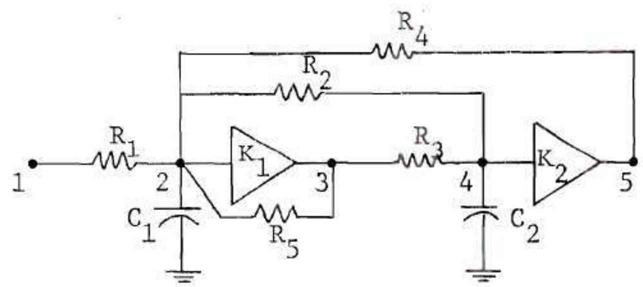
Figure	C_1	C_2	C_3	C_4	C_5	C_6
137	1	1	1	1	1	1
138	1	1	1	1	1	0
139	1	0	1	1	1	1
140	1	0	1	1	1	0



$K_1 = -K_0$

$K_2 = K_0$

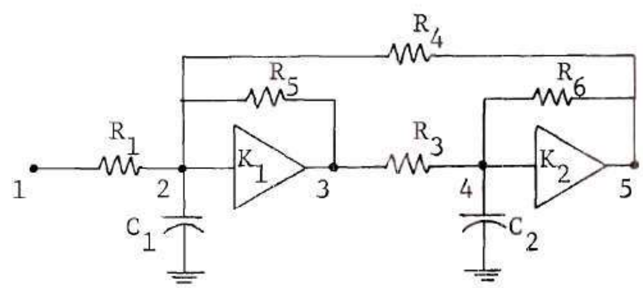
Figure 133. A Low-pass Filter Circuit



$K_1 = -K_0$

$K_2 = K_0$

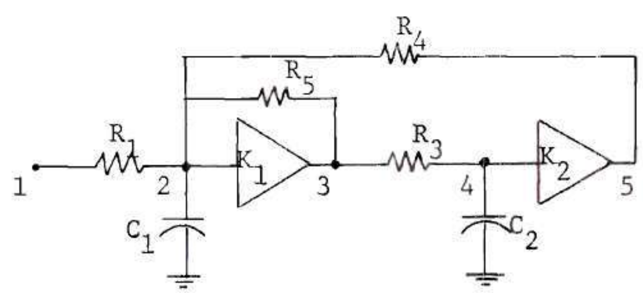
Figure 134. A Low-pass Filter Circuit



$K_1 = -K_0$

$K_2 = K_0$

Figure 135. A Low-pass Filter Circuit



$K_1 = -K_0$

$K_2 = K_0$

Figure 136. A Low-pass Filter Circuit

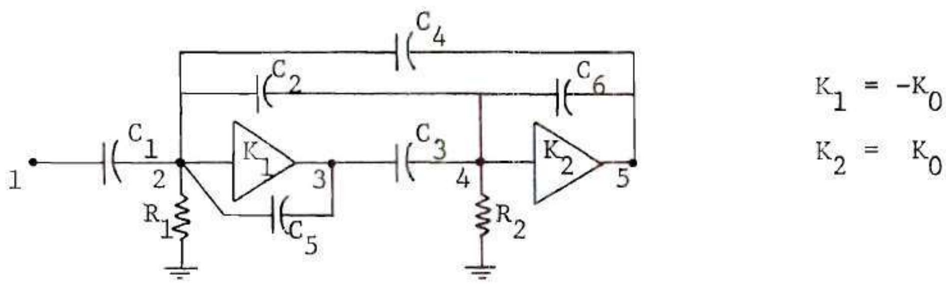


Figure 137. A High-pass Filter Circuit

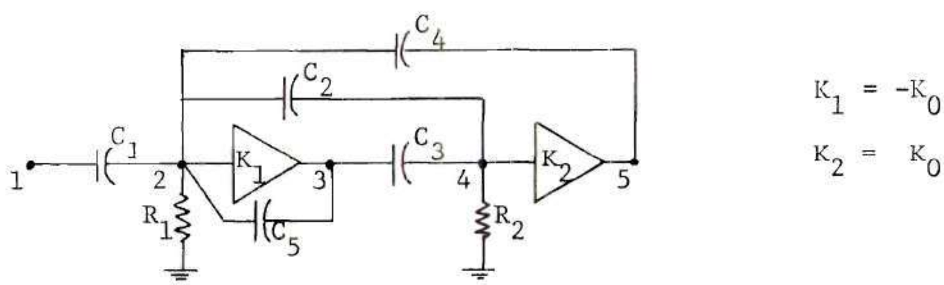


Figure 138. A High-pass Filter Circuit

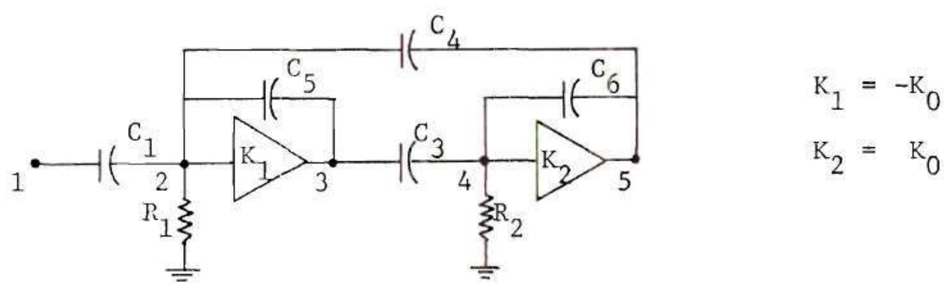


Figure 139. A High-pass Filter Circuit

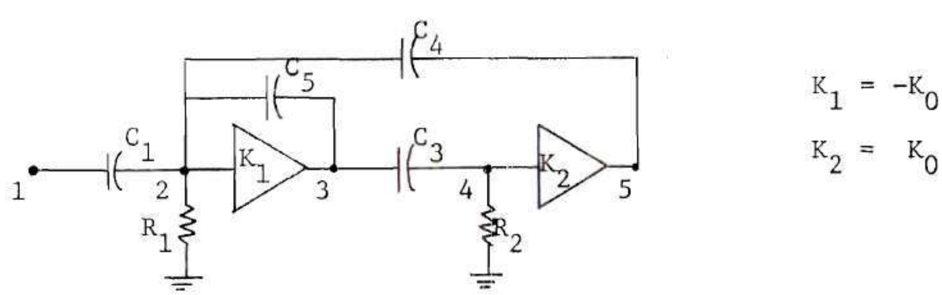


Figure 140. A High-pass Filter Circuit

$$+s\{G_1(C_2+C_3+C_6)+G_2(C_1+C_2+C_4+C_5)+K_0G_2C_5-K_0G_1C_6\}+G_1G_2$$

For the circuits of Figures 138 to 140, N(s) and D(s) can be found using equations (196), (197), and Table 14.

(iv) If we choose $K_1 = -K_0^2$, $K_2 = \frac{1}{K_0}$ and substitute into (128), then this equation and (177) will be the basic equations for realizing the filters. After searching all possible combinations of y's, six circuits as shown in Figures 141 to 146 are found possible for the low-pass sections. Their duals, Figures 147 to 152, have the characteristics of the high-pass sections.

It is most conveneint to present Figures 141 to 146 and Figures 147 to 152 by Tables 15 and 16, which show the basic circuit for each group, and show which resistors or capacitors are retained in each figure.

For the circuit of Figure 141:

$$N(s) = G_1(G_2+K_1G_3) \tag{198}$$

$$G_1(G_2-K_0^2G_3)$$

$$D(s) = s^2C_1(C_2+C_3)+s\{C_1(G_2+G_3+G_4)+(C_2+C_3)(G_1+G_2)-K_2C_3 \tag{199}$$

$$(G_1+G_2)-K_2C_1G_4-K_2C_1G_2-K_1K_2C_1G_3\}+G_1(G_2+G_3)$$

$$+G_4(G_1+G_2)(1-K_2)+G_2G_3(1-K_1)$$

$$= s^2C_1(C_2+C_3)+s\{C_1(G_2+G_3+G_4)+(C_2+C_3)(G_1+G_2)$$

$$-\frac{1}{K_0}\{C_3(G_1+G_2)+C_1G_4+C_1G_2\}+K_0C_1G_3\}+G_1(G_2+G_3)$$

$$+G_4(G_1+G_2)(1-\frac{1}{K_0})+G_2G_3(1+K_0^2)$$

For the circuits of Figures 142 to 146, N(s) and D(s) can be found by using equations (198), (199), and Table 15.

For the circuit of Figure 147:

$$N(s) = C_1(C_2-K_0^2C_3)s^2 \tag{200}$$

$$D(s) = s^2\{C_1(C_2+C_3)+C_4(C_1+C_2)(1-\frac{1}{K_0})+C_2C_3(1+K_0^2)\} \tag{201}$$

$$+s\{G_1(C_2+C_3+C_4)+(G_2+G_3)(C_1+C_2)-\frac{1}{K_0}\{G_3(C_1+C_2)$$

$$+G_1C_4+G_1C_2)+K_0G_1C_3\}+G_1(G_2+G_3)$$

For the circuits of Figures 148 to 152, N(s) and D(s) can be found by using equations (200), (201), and Table 16.

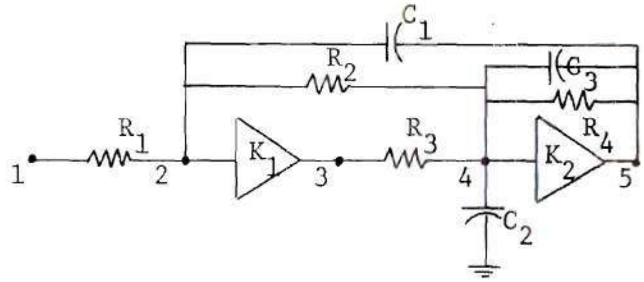
If the output is to be taken from node 2 instead of node 4, we can use the same procedures and obtain the same set of networks. This is equivalent to the interchanging of nodes 2 and 3 with nodes 4 and 5. Therefore, the low-pass and high-pass sections for which the outputs are taken from node 2 will not be shown here.

Table 15. The Relationships of Circuits of Figures 141 to 146

Figure	C_1	C_2	C_3	R_1	R_2	R_3	R_4
141	1	1	1	1	1	1	1
142	1	1	0	1	1	1	1
143	1	1	1	1	1	1	0
144	1	1	0	1	1	1	0
145	1	0	1	1	1	1	1
146	1	0	1	1	1	1	0

Table 16. The Relationships of Circuits of Figures 147 to 152

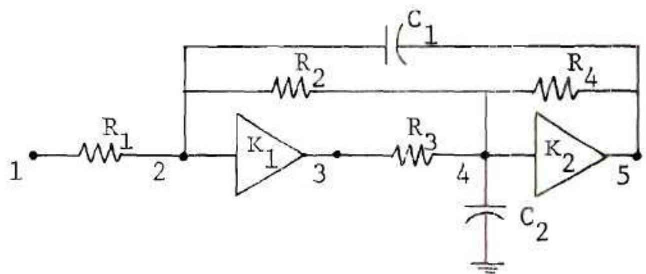
Figure	R_1	R_2	R_3	C_1	C_2	C_3	C_4
147	1	1	1	1	1	1	1
148	1	1	0	1	1	1	1
149	1	1	1	1	1	1	0
150	1	1	0	1	1	1	0
151	1	0	1	1	1	1	1
152	1	0	1	1	1	1	0



$K_1 = -K_0^2$

$K_2 = \frac{1}{K_0}$

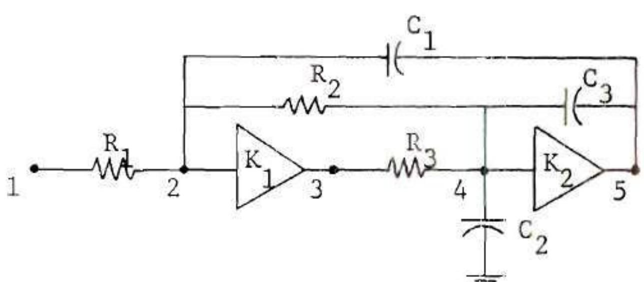
Figure 141. A Low-pass Filter Circuit



$K_1 = -K_0^2$

$K_2 = \frac{1}{K_0}$

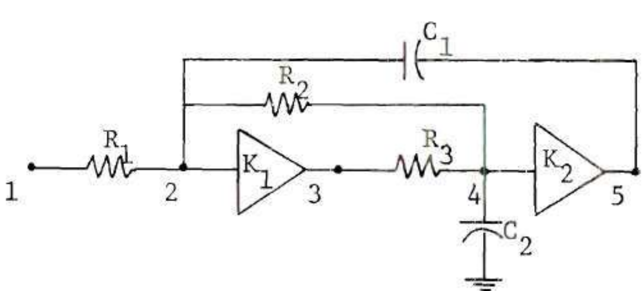
Figure 142. A Low-pass Filter Circuit



$K_1 = -K_0^2$

$K_2 = \frac{1}{K_0}$

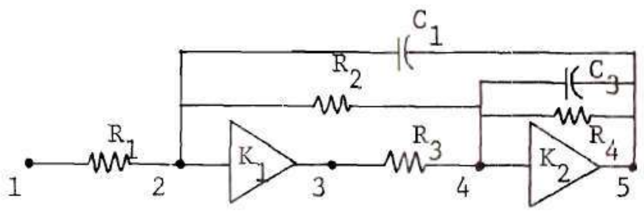
Figure 143. A Low-pass Filter Circuit



$K_1 = -K_0^2$

$K_2 = \frac{1}{K_0}$

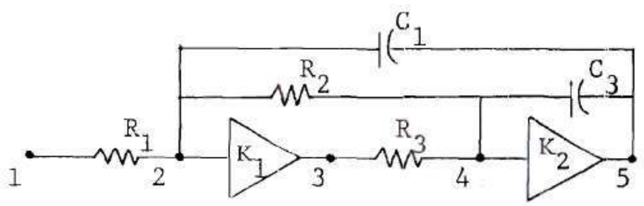
Figure 144. A Low-pass Filter Circuit



$K_1 = -K_0^2$

$K_2 = \frac{1}{K_0}$

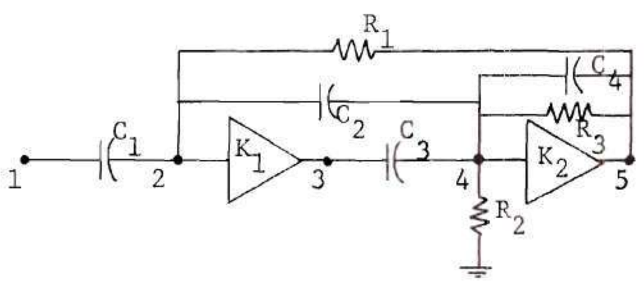
Figure 145. A Low-pass Filter Circuit



$K_1 = -K_0^2$

$K_2 = \frac{1}{K_0}$

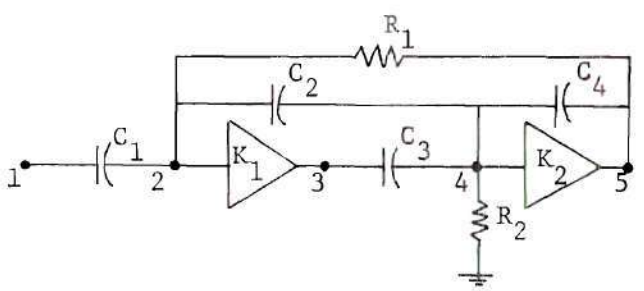
Figure 146. A Low-pass Filter Circuit



$K_1 = -K_0^2$

$K_2 = \frac{1}{K_0}$

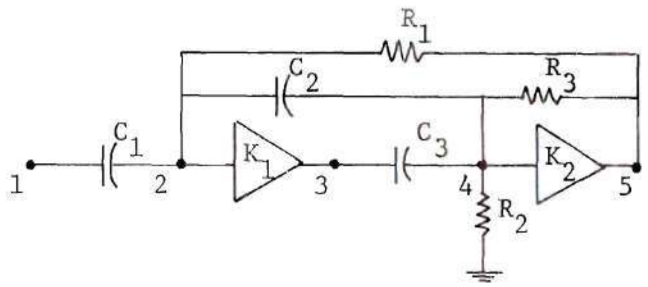
Figure 147. A High-pass Filter Circuit



$K_1 = -K_0^2$

$K_2 = \frac{1}{K_0}$

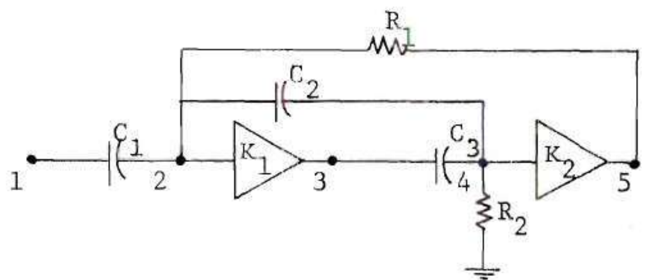
Figure 148. A High-pass Filter Circuit



$K_1 = -K_0^2$

$K_2 = \frac{1}{K_0}$

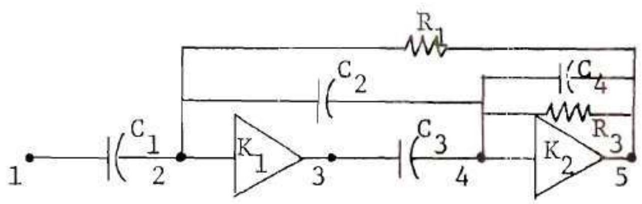
Figure 149. A High-pass Filter Circuit



$K_1 = -K_0^2$

$K_2 = \frac{1}{K_0}$

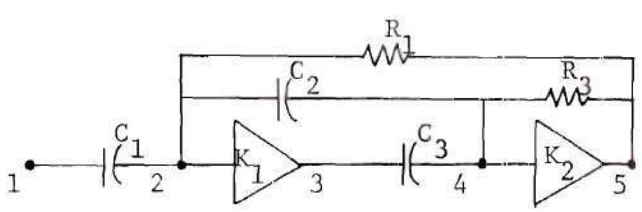
Figure 150. A High-pass Filter Circuit



$K_1 = -K_0^2$

$K_2 = \frac{1}{K_0}$

Figure 151. A High-pass Filter Circuit



$K_1 = -K_0^2$

$K_2 = \frac{1}{K_0}$

Figure 152. A High-pass Filter Circuit

By using the network configurations of Figures 5 and 6, and keeping the number of passive elements between each node and ground to be minimum, a total of thirty-three circuits are found possible for realizing the constant-Q, tunable low-pass sections. The same number of circuits are found possible for realizing the constant-Q, tunable high-pass sections.

Even though the number of total circuits are quite large, these circuits are derived from the basic circuits. The basic circuits for the low-pass sections are Figures 87, 99, 111, 112, 113, 125, 133, and 141. The basic circuits for the high-pass sections, which are the dual circuits of those for the low-pass sections, are Figures 89, 101, 118, 119, 120, 129, 137, and 147. These basic circuits have been realized without the use of computer programming.

CHAPTER V

THE MINIMIZATIONS OF THE CONTROLLED GAINS AND PASSIVE ELEMENTS

In Chapters III and IV we have derived the circuits for band-pass, low-pass, and high-pass sections with the approximation that the controlled gains are much larger than unity. In practice, finite values of these gains are used and results in slight changes in Q_0 due to changes in the values of the controlled gains.

When a bound on the changes in Q_0 is given, we can find the minimum required controlled gains such that the changes in these gains, for tuning the filters, will be within the given bound. If higher gains are used, the approximations will be improved; and the bound on the changes in Q_0 due to the changes in these gains will be smaller.

According to ANSI standards, constant-Q filters are classified as full-octave, half-octave, and third octave filter sets.[13] In each class the transmission losses are allowed to vary between an upper bound and a lower bound. This is equivalent to allowing the filters in a class a slight change in Q. Therefore, the nonideal constant-Q characteristics of the derived sections do not give us any difficulty since we can design each section to be of the same class throughout the tuning range.

Since the derived sections can be grouped according to their basic circuits, we will minimize required gains of the basic circuits and check whether these gains can be used for other circuits or not.

In this chapter, the minimizations on the number of passive elements will also be made; that is, using only the necessary passive elements.

For all circuits derived in Chapters III and IV, the products of the controlled gains are either K_0^2 or K_0. For example, we choose $K_1 = -K_0$ and $K_2 = K_0$. We then have $K_1K_2 = -K_0^2$. There is no advantage in letting the two gains be of different magnitudes. This is because when the magnitude of K_1 is increased, we have to decrease the magnitude of K_2 by the same factor.

To illustrate the minimization on the controlled gains and on the number of passive elements, the circuit of Figure 7 will be analyzed in detail. We have

$$K_1 = -K_0 < 0 \tag{202}$$

$$K_2 = K_0 \tag{203}$$

We let

$$C_1 = C < 0 \tag{204}$$

$$C_2 = bC, \; b > 0 \tag{205}$$

$$C_3 = dC, \; d > 0 \tag{206}$$

$$G_2(1+b)+(G_1+G_3)(1+d) = AK_0\big(d(G_1+G_3)+G_3\big), \tag{207}$$

$$A > 0$$

and use the approximation that

$$b(1+K_0^2)d(1+b)(1+K_0) \approx bK_0^2 \tag{208}$$

We have

$$\frac{V_3}{V_1} = \frac{G_1Cs}{s^2C^2K_0^2b+s(1+A)K_0C\left(d(G_1+G_3)+G_3\right)+G_2(G_1+G_3)} \tag{209}$$

$$\omega_0 = \sqrt{\frac{G_2(G_1+G_3)}{b}} \cdot \frac{1}{CK_0} \tag{210}$$

$$Q_0 = \frac{G_2(G_1+G_3)b}{(1+A)\left[(G_1+G_3)+G_3\right]} \tag{211}$$

$$A_0 = \frac{G_1}{(1+A)K_0\left[d(G_1+G_3)+G_3\right]} \tag{212}$$

We see that if $K_0 >> 1$, then from (207), A will be very small. Q_0 in (211) will be independent of K_0.

If the output is taken from terminal 4 instead of terminal 3, then

$$(A_0)_{V_4} = \frac{-G_1}{(1+A)\left[d(G_1+G_3)+G_3\right]} \tag{213}$$

Now we can formulate a set of equations for designing the filter capable of tuning the center-frequency from ω_0 to ω_1 by changing K_0 to KN and keeping Q_0 within a certain tolerance. Let

$$\omega_1 = N\,\omega_0\ ,\ N > 0 \tag{214}$$

$$K_N = \frac{K_0}{N} \tag{215}$$

We define the percentage changes in Q_0 and A_0 as

$$\frac{\Delta Q_1}{Q_0} = \frac{Q_1 - Q_0}{Q_0} \tag{216}$$

$$\frac{\Delta A_1}{A_0} = \frac{A_1 - A_0}{A_0} \tag{217}$$

$$Q_1 = \frac{G_2(G_1+G_3)b}{(1+NA)\left(d(G_1+G_3)+G_3\right)} \tag{218}$$

$$(A_1)_{V_4} = \frac{-G_1}{(1+NA)\left(d(G_1+G_3)+G_3\right)} \tag{219}$$

$$\frac{\Delta Q_1}{Q_0} = \left(\frac{\Delta A_1}{A_0}\right)_{V_4} = \frac{(1-N)A}{1+NA} \tag{220}$$

From equations (207) and (211),

$$Q_0 = \frac{G_2(G_1+G_3)b}{G_2(1+b)+G_1+G_3)(1+d)} \cdot \frac{AK_0}{1+A} \tag{221}$$

or

$$K_0 = Q_0 \cdot \frac{1+A}{A} \cdot \frac{G_2(1+b)+(G_1+G_3)(1+d)}{G_2(G_1+G_3)b} \tag{222}$$

From (222) it is easy to see that

$$K_{0_{min}} = 2Q_0 \frac{1+A}{A} \tag{223}$$

when

$$b >> 1 \quad \text{and} \quad d << 1 \tag{224}$$

and

$$G_2 b = G_1+G_3 \qquad (225$$

We note that (208) and (224) can be satisfied at the same time. For any finite $d > 0$, we have

$$K_0 = (2+d)Q_0 \frac{1+A}{A} \tag{226}$$

The circuit of Figure 9, which is derived from this circuit, should be used since $C_3 = 0$ or $d = 0$. Then $K_{0_{min}}$ can be used and, moreover, the band-pass section is realized with one less capacitor.

The minimization for the rest of the derived sections can be made in the similar manner. When the center-frequency $\omega_1 = N\,\omega_0$, $N > 0$ as in (214), the controlled gains will be changed from K_0 to K_N.

$$\text{If } \omega_0 \;\alpha\; \frac{1}{K_0},\ \text{then } K_N = \frac{K_0}{N} \tag{227}$$

$$\text{If } \omega_0 \;\alpha\; K_0,\ \text{then } K_N = NK_0 \tag{228}$$

Tables 17, 18, and 19 summarize the results of the minimization in the controlled gains and in the number of the passive elements for the band-pass, the low-pass, and the high-pass sections.

The first column of each table shows the circuits that are grouped together. All circuits in each group are derived from their basic circuit which is shown as the first circuit of each group.

The second column of each table shows the best circuit for each group. The chosen circuit of each group utilizes the minimum controlled gains and the least number of passive elements within the same group.

Other columns show the values of various quantities and certain conditions to be met for designing the chosen circuits. Outputs are taken from terminal 4 unless otherwise indicated. If certain approximations in the tables cannot be met, we can also design the filtering section by referring to the exact formulae as shown in Chapters III and IV.

The fourth column of each table shows the smallest value of K_0 that is needed for each chosen circuit so that $\Delta Q_1/Q_0$ will be within the given tolerance throughout the tuning range. Larger K_0 can always be used, and we can expect the circuit to have smaller $\Delta Q_1/Q_0$. For many circuits we have $\Delta Q_1/Q_0$ equal to $\Delta A_1/A_0$. This means that the percentage change in the gains at the center-frequency is equal to the percentage change in Q_0 for that circuit.

For the circuits shown in Figures 15 to 22, Figures 35 to 38, Figures 91 to 94, and Figures 103 to 106, the analyses will be the same as those shown in Figure 7 to 14, Figures 31 to 34, Figures 87 to 90, and Figures 99 to 102, respectively. The difference is in the choices of K's; that is, $K_1 = -K_0^2$ and $K_2 = 1/K_0$ instead of $K_1 = -K_0$ and $K_2 = K_0$. For all circuits, K_1 will be very large and K_2 will be very small. When tuning, if K_1 is increasing K_2 must be decreasing; however, there is no advantage in doing this since it is easier to increase or decrease K_1 and K_2 identically.

For the circuits shown in Figures 23 to 30, Figures 39 to 42, Figures 95 to 98, and Figures 107 to 110, the analyses will be the same as those shown in Figures 7 to 14, Figures 31 to 34, Figures 87 to 90, and Figures 99 to 102, respectively. The difference is also in the choices of K's; that is, $K_1 = -K_0$ and $K_2 = -K_0^2$ instead of $K_1 = -K_0$ and $K_2 = K_0$. Since the circuits for the two choices of K's are essentially identical, again there is no advantage in using $K_2 = -K_0^2$ when we can use $K_2 = K_0$.

The circuits shown in Figures 43 to 60 and Figures 111 to 124 are not useful since Q_0 of each circuit is smaller than one-half. Therefore, none of these circuits is shown in Tables 17, 18, and 19.

Fourteen useful circuits are chosen for the band-pass sections. Six each are chosen for the low-pass and the high-pass sections.

Table 17. The Optimum Band-pass Sections and Their Specifications

Circuits of Figures	Best Circuit	ω_0	K_0	A_0	$\frac{\Delta Q_1}{Q_0}$	$\frac{\Delta A_1}{A_0}$	Remarks
7,9	9	$\frac{G_2}{CK_0}$	$2Q_0\frac{1+A}{A}$	$\frac{-G_1}{G_3(1+A)}$	$\frac{(1-N)A}{1+NA}$	$\frac{(1-N)A}{1+NA}$	$K_1 = -K_2 = -K_0<0$ $C_1 = C>0,\ C_2 = bC$ $G_2b = G_3Q_0(1+A),\ b>>1$ $G_2b = G_1 + G_3$
8,10	10	$\frac{G_2}{CK_0}$	$(2+b)Q_0\frac{1+A}{A}$	$\frac{-1}{(1+A)b}$	$\frac{(1-N)A}{1+NA}$	$\frac{(1-N)A}{1+NA}$	$K_1 = -K_2 = -K_0<0$ $C_1 = C>0,\ C_2=bC,\ C_4=eC$ $G_1 = G_2e,\ e>>1$ $Q_0 = \frac{1}{(1+A)b}$
11,13	13	$\frac{GK_0}{C_2}$	$2Q_0\frac{1+A}{A}$	$\frac{-C_1}{C_3(1+A}$	$\frac{A(N-1)}{N+A}$	$\frac{A(N-1)}{N+A}$	$K_1 = -K_2 = -K_0<0$ $G_1 = G>0,\ G_2 = bG$ $C_2b = C_3Q_0(1+A),\ b>>1$ $C_2b = C_1 + C_3$
12,14	14	$\frac{GK_0}{C_2}$	$(2+b)Q_0\frac{1+A}{A}$	$\frac{-1}{(1+A)b}$	$\frac{A(N-1)}{N+A}$	$\frac{A(N-1)}{N+A}$	$K_1 = -K_2 = -K_0<0$ $G_1 = G>0,\ G_2=bG,\ G_4=eG$ $C_1 = C_2e,\ e>>1$ $Q_0 = \frac{1}{(1+A)b}$

Circuits of Figures	Best Circuit	ω_0	K_0	A_0	$\frac{\Delta Q_1}{Q_0}$	$\frac{\Delta A_1}{A_0}$	Remarks
31, 32	32*	$\frac{G_1}{CK_0}$	$(2+d)Q_0\frac{1+A}{A}$	$\frac{-K_0}{(1+A)d}$	$\frac{(1-N)A}{1+NA}$	$\frac{1+A}{N(1+NA)}-1$	$K_1 = -K_2 = -K_0<0$ $C_1 = C>0,\ C_2=bC,\ C_3=dC$ $G_1 b = G_2,\ b>>1$ $Q_0 = \frac{1}{(1+A)d}$
33, 34	34*	$\frac{GK_0}{C_1}$	$(2+d)Q_0\frac{1+A}{A}$	$\frac{-K_0}{(1+A)d}$	$\frac{A(N-1)}{N+A}$	$\frac{N^2(1+A)}{N+A}-1$	$K_1 = -K_2 = -K_0<0$ $G_1 = G>0,\ G_2=bG,\ G_3=dG$ $C_1 b = C_2,\ b>>1$ $Q_0 = \frac{1}{(1+A)d}$
61, 62, 63, 64	64	$\frac{G_2}{CK_0}$	$(2+d)Q_0\frac{1+A}{A}$	$\frac{1}{(1+A)d}$	$\frac{(1-N)A}{1+NA}$	$\frac{(1-N)A}{1+NA}$	$K_1 = -K_2 = K_0>0$ $C_1 = C>0,\ C_3=dC,\ C_5=fC$ $G_1 = G_2 f,\ f>>1$ $Q_0 = \frac{1}{(1+A)d}$
65, 66, 67, 68	68	$\frac{GK_0}{C_2}$	$(2+d)Q_0\frac{1+A}{A}$	$\frac{1}{(1+A)d}$	$\frac{A(N-1)}{N+A}$	$\frac{A(N-1)}{N+A}$	$K_1 = -K_2 = K_0>0$ $G_1 = G>0,\ G_3=dG,\ G_5=fG$ $C_1 = C_2 f,\ f>>1$ $Q_0 = \frac{1}{(1+A)d}$

Circuits of Figures	Best Circuit	ω_0	K_0	A_0	$\frac{\Delta Q_1}{Q_0}$	$\frac{\Delta A_1}{A_0}$	Remarks
69,70, 71,72	72	$\frac{G_2}{CK_0}$	$2Q_0\frac{1+A}{A}+\frac{1}{A}$	$\frac{-f}{(1+A)b}$	$\frac{(1-N)A}{1+NA}$	$\frac{(1-N)A}{1+NA}$	$K_1=-K_2=-K_0<0$ $C_1=C>0,\ C_2=bC,\ C_5=fC$ $G_1=G_2f,\ f>>1,\ b>1$ $Q_0=\frac{f}{(1+A)b}$
73,74, 75,76	76	$\frac{GK_0}{C_2}$	$2Q_0\frac{1+A}{A}+\frac{1}{A}$	$\frac{-f}{(1+A)b}$	$\frac{A(N-1)}{N+A}$	$\frac{A(N-1)}{N+A}$	$K_1=-K_2=-K_0<0$ $G_1=G>0,\ G_2=bG,\ G_5=fG$ $C_1=C_2f,\ f>>1,\ b>1$ $Q_0=\frac{f}{(1+A)b}$
77,78, 79, 80,81	79**	$\frac{G_4}{CK_0}$	$2Q_0\frac{1+A}{A}$	$\frac{-G_1}{(1+A)G_3}$	$\frac{(1-N)A}{1+NA}$	$\frac{(1-N)A}{1+NA}$	$K_1=-K_0^2,\ K_2=\frac{1}{K_0},\ K>0$ $C_1=C>0,\ C_2=bC$ $G_1+G_3=bG_4,\ b<<1$ $Q_0=\frac{G_1+G_3}{G_3(1+A)}$
	81**	$\frac{G_2}{CK_0}$	$2Q_0\frac{1+A}{A}$	$\frac{-G_1}{(1+A)G_3}$	$\frac{(1-N)A}{1+NA}$	$\frac{(1-N)A}{1+NA}$	$K_1=-K_0^2,\ K_2=\frac{1}{K_0},\ K>0$ $C_1=C>0,\ C_2=bC$ $G_1+G_3=bG_2,\ b<<1$ $Q_0=\frac{G_1+G_3}{G_3(1+A)}$

Circuits of Figures	Best Circuit	ω_0	K_0	A_0	$\frac{\Delta Q_1}{Q_0}$	$\frac{\Delta A_1}{A_0}$	Remarks
82, 83, 84, 85, 86	84**	$\frac{GK_0}{C_4}$	$2Q_0\frac{1+A}{A}$	$\frac{-C_1}{(1+A)C_3}$	$\frac{A(N-1)}{N+A}$	$\frac{A(N-1)}{N+A}$	$K_1 = -K_0^2,\ K_2=\frac{1}{K_0},\ K_0>0$ $G_1 = G>0,\ G_2 = bG$ $C_1+C_3 = bC_4,\ b<<1$ $Q_0 = \frac{C_1+C_3}{C_3(1+A)}$
	86**	$\frac{GK_0}{C_2}$	$2Q_0\frac{1+A}{A}$	$\frac{-C_1}{(1+A)C_3}$	$\frac{A(N-1)}{N+A}$	$\frac{A(N-1)}{N+A}$	$K_1 = -K_0^2,\ K_2=\frac{1}{K_0},\ K_0>0$ $G_1 = G>0,\ G_2=bG$ $C_1+C_3 = bC_2,\ b<<1$ $Q_0 = \frac{C_1+C_3}{C_3(1+A)}$

*The output is taken from terminal 2

**The output is taken from terminal 5

Table 18. The Optimum Low-pass Sections and Their Specifications

Circuits of Figures	Best Circuit	ω_0	K_0	$\lim_{s\to 0} A_0(s)$	$\frac{\Delta Q_1}{Q_0}$	$\frac{\Delta A_1}{A_0}$	Remarks
87,88	88**	$\frac{d}{MC_1}GK_0$	$\frac{1+M^2(1+\frac{1}{d})}{M}Q_0\frac{1+A}{A}$	$-\frac{1}{d}$	$\frac{A(N-1)}{N+A}$	$\frac{A(N-1)}{N+A}$	$K_1 = -K_2=-K_0<0$ $G_1 = G>0,\ G_2=bG,\ G_3=dG$ $C_2d = M^2C_1b,\ M>0,\ d>>b$ $Q_0 = \frac{M}{(1+A)}$
99,100	100*	$\frac{eGK_0}{C_2}$	$(2+\frac{b}{e})Q_0\frac{1+A}{A}$	$-\frac{1}{b}$	$\frac{A(N-1)}{N+A}$	$\frac{A(N-1)}{N+A}$	$K_1 = -K_2=-K_0<0$ $G_1 = G>0,\ G_2=bG,\ G_4=eG$ $G_5 = fG$ $C_1e = C_2b,\ b>>1+f$ $Q_0 = \frac{e}{(1+A)f}$
125,126, 127,128	128**	$\frac{eGK_0}{C_1}$	$2Q_0\frac{1+A}{A}+\frac{1}{A}$	$-\frac{1}{e}$	$\frac{A(N-1)}{N+A}$	$\frac{A(N-1)}{N+A}$	$K_1 = -K_2=K_0>0$ $G_1 = G>0,\ G_3=dG$ $G_4 = eG,\ G_6=gG$ $C_1d = C_2e,\ e>>1$ $Q_0 = \frac{d}{g(1+A)}$

Circuits of Figures	Best Circuit	ω_0	K_0	$\lim_{s\to 0} A_0(s)$	$\frac{\Delta Q_1}{Q_0}$	$\frac{\Delta A_1}{A_0}$	Remarks
133,134, 135,136	136**	$\frac{eGK_0}{C_1}$	$2Q_0\frac{1+A}{A}+\frac{1}{A}$	$-\frac{1}{e}$	$\frac{A(N-1)}{N+A}$	$\frac{A(N-1)}{N+A}$	$K_1 = -K_2=-K_0<0$ $G_1 = G>0,\ G_3=dG$ $G_4 = eG,\ G_5=fG$ $C_1d = C_2e,\ e>>1$ $Q_0 = \frac{e}{f(1+A)}$
141,142, 143,144, 145,146	144	$\frac{bGK_0}{MC_1}$	$\frac{1+M^2}{M}Q_0\frac{1+A}{A}$	$-\frac{1}{b}$	$\frac{A(N-1)}{N+A}$	$\frac{A(N-1)}{N+A}$	$K_1 = -K_0^2, K_2=\frac{1}{K_0}, K_0>0$ $G_1 = G>0,\ G_2=bG,\ G_3=dG$ $C_2b = M^2C_1d,\ M>0,\ d>>b>>1$ $Q_0 = \frac{M}{1+A}$
	146	$\frac{bGK_0}{MC_1}$	$\frac{1+M^2}{M}Q_0\frac{1+A}{A}$	$-\frac{1}{b}$	$\frac{A(N-1)}{N+A}$	$\frac{A(N-1)}{N+A}$	$K_1 = -K_0^2,\ K_2=\frac{1}{K_0},\ K_0>0$ $G_1 = G>0,\ G_2=bG,\ G_3=dG$ $C_3b = M^2C_1d,\ M>0,\ d>>b>>1$ $Q_0 = \frac{M}{1+A}$

*The output is taken from terminal 2

**The output is taken from terminal 5

Table 19. The Optimum High-pass Sections and Their Specifications

Circuits of Figures	Best Circuit	ω_0	K_0	$\lim\limits_{s\to\infty} A_0(s)$	$\frac{\Delta Q_1}{Q_0}$	$\frac{\Delta A_1}{A_0}$	Remarks
89,90	90^{**}	$\frac{MG_1}{dCK_0}$	$\frac{1+M^2(1+\frac{1}{d})}{M}Q_0\frac{1+A}{A}$	$-\frac{1}{d}$	$\frac{(1-N)A}{1+NA}$	$\frac{(1-N)A}{1+NA}$	$K_1 = -K_2 = -K_0 < 0$ $C_1 = C > 0,\ C_2 = bC,\ C_3 = dC$ $G_2 d = M^2 C_1 b,\ M > 0,\ d >> b$ $Q_0 = \frac{M}{1+A}$
101,102	102^{*}	$\frac{G_2}{eCK_0}$	$(2+\frac{b}{e})Q_0\frac{1+A}{A}$	$-\frac{1}{b}$	$\frac{(1-N)A}{1+NA}$	$\frac{(1-N)A}{1+NA}$	$K_1 = -K_2 = -K_0 < 0$ $C_1 = C > 0,\ C_2 = bC$ $C_4 = eC,\ C_5 = fC$ $G_1 e = G_2 b,\ b >> 1+f$ $Q_0 = \frac{e}{(1+A)f}$
129,130, 131,132	132^{**}	$\frac{G_1}{eCK_0}$	$2Q_0\frac{1+A}{A} + \frac{1}{A}$	$-\frac{1}{e}$	$\frac{(1-N)A}{1+NA}$	$\frac{(1-N)A}{1+NA}$	$K_1 = -K_2 = K_0 > 0$ $C_1 = C > 0,\ C_3 = dC$ $C_4 = eC,\ C_6 = gC$ $G_1 d = G_2 e,\ e >> 1$ $Q_0 = \frac{d}{g(1+A)}$

Circuits of Figures	Best Cir-Cuit	ω_0	K_0	$\lim\limits_{s\to\infty} A_0(s)$	$\frac{\Delta Q_1}{Q_0}$	$\frac{\Delta A_1}{A_0}$	Remarks
137,138, 139,140	140**	$\frac{G_1}{eCK_0}$	$2Q_0\frac{1+A}{A}+\frac{1}{A}$	$-\frac{1}{e}$	$\frac{(1-N)A}{1+NA}$	$\frac{(1-N)A}{1+NA}$	$K_1 = -K_2=-K_0<0$ $C_1 = C>0,\ C_3=dC$ $C_4 = eC,\ C_5=fC$ $G_1d = G_2e,\ e>>1$ $Q_0 = \frac{e}{f(1+A)}$
147,148, 149,150, 151,152	150	$\frac{MG_1}{bCK_0}$	$\frac{1+M^2}{M}Q_0\frac{1+A}{A}$	$-\frac{1}{b}$	$\frac{(1-N)A}{1+NA}$	$\frac{(1-N)A}{1+NA}$	$K_1 = -K_0^2, K_2=\frac{1}{K_0}, K_0>0$ $C_1 = C>0,\ C_2=bC,\ C_3=dC$ $G_2b = M^2G_1d,\ M>0,\ d>>b>>1$ $Q_0 = \frac{M}{1+A}$
	152	$\frac{MG_1}{bCK_0}$	$\frac{1+M^2}{M}Q_0\frac{1+A}{A}$	$-\frac{1}{b}$	$\frac{(1-N)A}{1+NA}$	$\frac{(1-N)A}{1+NA}$	$K_1 = -K_0^2,\ K_2=\frac{1}{K_0}, K_0>0$ $C_1 = C>0,\ C_2=bC,\ C_3=dC$ $G_3b = M^2G_1d,\ M>0,\ d>>b>>1$ $Q_0 = \frac{M}{1+A}$

*The output is taken from terminal 2

**The output is taken from terminal 5

CHAPTER VI

THE INPUT AND OUTPUT IMPEDANCES

In Chapter V we have dealt with the minimization of the controlled gains and the number of passive elements for the derived sections. Fourteen circuits are chosen for the band-pass sections, six circuits are chosen for the low-pass sections, and six circuits are chosen for the high-pass sections.

In order to utilize the derived sections to the greatest advantage, it is useful to know the input and output impedances of these circuits. After these input and output impedances are found, we can then choose the particular one that is most suitable for each application.

Since not all the derived circuits but only the twenty-eight chosen circuits will be used for designing, we will find the input and output impedances only for these chosen circuits.

Input Impedances

To find the input impedances of various filter sections, the

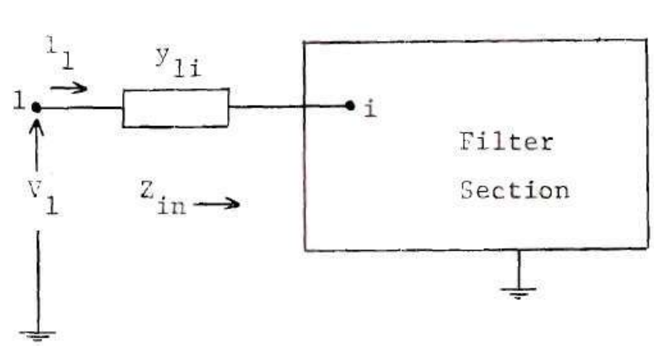


Figure 153. The Circuit Configuration for Finding the Input Impedance

arrangement as shown in Figure 153 is used. We have

$$I_1 = (V_1 - V_i)y_{1i} \tag{229}$$

$$\frac{V_i}{V_1} = \frac{N'(s)}{D(s)} \tag{230}$$

$$Z_{in} = \frac{V_1}{I_1} \tag{231}$$

$$= \frac{V_1}{V_1 - V_i} \cdot \frac{1}{y_{1i}}$$

$$= \frac{D(s)}{D(s) - N'(s)} \cdot \frac{1}{y_{1i}}$$

Output Impedances

Many circuits derived in Chapters III and IV are such that the outputs are taken from the outputs of the VCVS's. If the VCVS is assumed to be ideal, then the output impedances of these circuits are zero.

To find the output impedances of the circuits whose outputs are taken from the terminals that are not the outputs of the VCVS's, we will ground terminal 1 and use the ratio of a voltage applied at the output terminal to the corresponding current going into output terminal.

When terminal 1 is grounded, we have only two independent voltages for the other four terminals since we use two VCVS in our circuits. For the circuit configuration of Figure 5 with the output taken from terminal 2, we have

$$\begin{bmatrix} y_{22} & y_{23}+K_1y_{24}+K_1K_2y_{25} \\ y_{32} & y_{33}+K_1y_{34}+K_1K_2y_{35} \end{bmatrix} \begin{bmatrix} V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} I_2 \\ 0 \end{bmatrix} \tag{232}$$

In general, we have

$$\begin{bmatrix} y_{aa} & y_{ab} \\ y_{ba} & y_{bb} \end{bmatrix} \begin{bmatrix} V_a \\ V_b \end{bmatrix} = \begin{bmatrix} 0 \\ I_b \end{bmatrix} \tag{233}$$

where V_b = the output voltage

I_b = the output current

V_a = another independent voltage

$$Z_{in} = \frac{V_b}{I_b} \tag{234}$$

$$= \frac{y_{aa}}{y_{aa}y_{bb}-y_{ab}y_{ba}}$$

$$= \frac{y_{aa}}{D(s)}$$

D(s) in (234) is the same D(s) shown in Chapters III and IV.

The approximations of Tables 17, 18, and 19 are used together with (231) and (234) to find the input and output impedances. Tables 20, 21, and 22 show the input impedances for the chosen band-pass, low-pass, and high-pass sections. Tables 23, 24, and 25 show the output impedances for the chosen band-pass, low-pass, and high-pass sections.

When the approximations of Tables 17, 18, and 19 are used, the input and output impedances of the circuits of each group are practically equal to those of the chosen circuits. This shows that we should use the chosen circuits to design the sections.

The expression for $Z_{in}(s)$ and $Z_{out}(s)$ are very complicated and so are $Z_{in}(j\omega_0)$ and $Z_{out}(j\omega_0)$. Therefore, only $Z_{in}(s)$ and $Z_{out}(s)$ are shown in Tables 20 to 25. Limits as s approaches zero and infinity are shown. For example, from Table 20, the input impedance of Figure 9 behaves like a resistance of $Q_0(1+A)R_1$ at low frequency and R_1 at high frequency; but the input impedance of Figure 10 behaves like a capacitance of $(AK_0b-1)CR_2/R_1$ at low frequency and like a resistance of R_1 at high frequency.

From (231), $|Z_{in}| \geq 1/|y_{1i}|$ as can be seen from the structure of the circuit. The results in the tables agree with (231).

Table 20. The Input Impedances of the Optimum Band-pass Sections

Circuit of Figure	$N'(s)$	$Z_{in}(s)$	$\lim_{s\to 0} Z_{in}(s)$	$\lim_{s\to\infty} Z_{in}(s)$
9	$G_1(C_1s+G_2)$	$\dfrac{s^2C^2K_0^2b+s(1+A)K_0CG_3+G_2(G_1+G_3)}{s^2C^2K_0^2b+s(1+\frac{A}{2})K_0CG_3+G_2G_3}R_1$	$Q_0(1+A)R_1$	R_1
10	$G_1(C_1s+G_2K_0C_2s)$	$\dfrac{s^2C^2K_0^2e+s(1+A)K_0G_1Cb+G_1G_2}{s^2C^2K_0^2e+s(AK_0b-1)G_1C}R_1$	$\dfrac{R_1}{s(AK_0b-1)CR_2}$	R_1
13	$C_1s(G_1+C_2s)$	$\dfrac{s^2C_2(C_1+C_3)+s(1+A)K_0GC_3+G^2K_0^2b}{s^2C_2C_3+s(1+\frac{A}{2})K_0GC_3+G^2K_0^2b}\cdot\dfrac{1}{C_1s}$	$\dfrac{1}{C_1s}$	$\dfrac{Q_0(1+A)}{C_1s}$
14	$C_1s(G_1+C_2s+K_0G_2)$	$\dfrac{s^2C_1C_2+s(1+A)K_0C_1Gb+G^2K_0^2e}{S(AK_0b-1)GC_1+G^2K_0^2e}\cdot\dfrac{1}{C_1s}$	$\dfrac{1}{C_1s}$	$\dfrac{C_2}{(AK_0b-1)GC_1}$
32	$G_1(G_2+C_1s+C_2s)$	$\dfrac{s^2C^2K_0^2b+s(1+A)K_0G_2Cd+G_1G_2}{S^2C^2K_0^2b+s\left(G_2(1+d)+K_0G_2d\right)C}R_1$	$\dfrac{1}{sC(1+K_0d)}$	R_1
34	$C_1s(C_2s+G_1+G_2)$	$\dfrac{s^2C_1C_2+s(1+A)K_0C_2Gd+G^2K_0^2b}{sG\left(C_2(1+d)+K_0C_2d\right)+G^2K_0^2b}\cdot\dfrac{1}{C_1s}$	$\dfrac{1}{C_1s}$	$\dfrac{1}{G(1+d+K_0d)}$

Circuit of Figure	$N'(s)$	$Z_{in}(s)$	$\lim_{s\to 0} Z_{in}(s)$	$\lim_{s\to\infty} Z_{in}(s)$
64	$G_1(G_2+C_1s+C_3s+K_0C_3s)$	$\frac{s^2C^2K_0^2f+sC(1+A)K_0G_1d+G_1G_2}{s^2C^2K_0^2f+sCG_2f}\cdot R_1$	$\frac{1}{sCf}$	R_1
68	$C_1s(C_2s+G_1+G_3+K_0G_3)$	$\frac{s^2C_1C_2+sG(1+A)K_0C_1d+G^2K_0^2f}{sGC_2f+G^2K_0^2f}\cdot\frac{1}{C_1s}$	$\frac{1}{C_1s}$	$\frac{1}{Gf}$
72	$G_1(G_2+C_1s)$	$\frac{s^2C^2K_0^2f+s(1+A)K_0G_2Cb+G_1G_2}{s^2C^2K_0^2f+s[(1+A)K_0G_2Cb-G_1C]}\cdot R_1$	$\frac{G_2}{s[(1+A)K_0G_2Cb-G_1C]}$	R_1
76	$C_1s(C_2s+G_1)$	$\frac{s^2C_1C_2+s(1+A)K_0C_2Gb+G^2K_0^2b}{s[(1+A)K_0C_2Gb-C_1G]+G^2K_0^2b}\cdot\frac{1}{C_1s}$	$\frac{1}{C_1s}$	$\frac{C_2}{(1+A)K_0G_2Cb-G_1C}$
79	$G_1(C_1s+C_2s+\frac{G_4}{K_0})$	$\frac{s^2C^2K_0^2b+s(1+A)K_0CG_3+(G_1+G_3)G_4}{s^2C^2K_0^2b+sC[(1+A)K_0\ G_3-G_1(1+b)]+(G_1+G_3)G_4}\cdot R_1$	R_1	R_1
81	$G_1(C_1s+C_2s+G_2)$	$\frac{s^2C^2K_0^2b+s(1+A)K_0CG_3+(G_1+G_3)G_2}{s^2C^2K_0^2b+s[(1+A)K_0CG_3-G_1C(1+b)]+G_2G_3}\cdot R_1$	$Q_0(1+A)R_1$	R_1

Circuit of Figure	$N'(s)$	$Z_{in}(s)$	$\lim_{s\to 0} Z_{in}(s)$	$\lim_{s\to\infty} Z_{in}(s)$
84	$C_1s(G_1+G_2+\frac{1}{K_0}C_4s)$	$\frac{s^2(C_1+C_3)C_4+s(1+A)K_0GC_3+G^2K_0^2b}{s^2(C_1+C_3)C_4+sG[(1+A)K_0C_3-C_1(1+b)]+G^2K_0^2b}\frac{1}{C_1s}$	$\frac{1}{C_1s}$	$\frac{1}{C_1s}$
86	$C_1s(G_1+G_2+C_2s)$	$\frac{s^2(C_1+C_3)C_2+s(1+A)K_0GC_3+G^2K_0^2b}{s^2C_2C_3+s[(1+A)K_0GC_3-C_1G(1+b)]+G^2K_0^2b}\frac{1}{C_1s}$	$\frac{1}{C_1s}$	$\frac{Q_0(1+A)}{C_1s}$

Table 21. The Input Impedances of the Optimum Low-pass Sections

Circuit of Figure	$N'(s)$	$Z_{in}(s)$	$\lim_{s\to 0} Z_{in}(s)$	$\lim_{s\to\infty} Z_{in}(s)$
88	$G_1(G_2+C_2s)$	$\dfrac{s^2C_1C_2+s(1+A)K_0C_1Gb+G^2K_0^2bd}{s^2C_1C_2+sG[(1+A)K_0C_1b-C_2]+G^2K^2bd}R_1$	R_1	R_1
100	$G_1(G_2+G_4+C_2s)$	$\dfrac{s^2C_1C_2+s(1+A)K_0C_2Gf+G^2K_0^2be}{s^2C_1C_2+sG[(1+A)K_0C_2f-C_2]+G^2K_0^2be}R_1$	R_1	R_1
128	$G_1(G_3+G_6+C_2s+K_0G_6)$	$\dfrac{s^2C_1C_2+s(1+A)K_0C_1Gg+G^2K_0^2de}{s^2C_1C_2+sG[(1+A)K_0C_1g-C_2]+G^2K_0^2de}R_1$	R_1	R_1
136	$G_1(G_3+C_2s)$	$\dfrac{s^2C_1C_2+s(1+A)K_0C_2Gf+G^2K_0^2de}{s^2C_1C_2+sG[(1+A)K_0C_2f-C_2]+G^2K_0^2de}R_1$	R_1	R_1
144	$G_1(G_2+G_3+C_2s)$	$\dfrac{s^2C_1C_2+s(1+A)K_0C_1Gd+G^2K_0^2bd}{s^2C_1C_2+sG[(1+A)K_0C_1d-C_2]+G^2K_0^2bd}R_1$	R_1	R_1
146	$G_1(G_2+G_3+C_3s-\frac{1}{K_0}C_3s)$	$\dfrac{s^2C_1C_3+s(1+A)K_0C_1Gd+G^2K_0^2bd}{s^2C_1C_2+sG[(1+A)K_0C_1d-C_3]+G^2K_0^2bd}R_1$	R_1	R_1

Table 22. The Input Impedances of the Optimum High-pass Sections

Circuit of Figure	$N'(s)$	$Z_{in}(s)$	$\lim_{s\to 0} Z_{in}(s)$	$\lim_{s\to\infty} Z_{in}(s)$
90	$C_1s(C_2s+G_2)$	$\frac{s^2C^2K_0^2bd+s(1+A)K_0G_1Cb+G_1G_2}{s^2C^2K_0^2bd+sC[(1+A)K_0G_1b-G_2]+G_1G_2}\ \frac{1}{C_1s}$	$\frac{1}{C_1s}$	$\frac{1}{C_1s}$
102	$C_1s(C_2s+C_4s+G_2)$	$\frac{s^2C^2K_0^2be+s(1+A)K_0G_2Cf+G_1G_2}{s^2C^2K_0^2be+sC[(1+A)K_0G_2f-G_2]+G_1G_2}\ \frac{1}{C_1s}$	$\frac{1}{C_1s}$	$\frac{1}{C_1s}$
132	$C_1s(C_3s+C_6s+G_2+K_0C_2s)$	$\frac{s^2C^2K_0^2de+s(1+A)K_0G_1Cg+G_1G_2}{s^2C^2K_0^2de+sC[(1+A)K_0G_1g-G_2]+G_1G_2}\ \frac{1}{C_1s}$	$\frac{1}{C_1s}$	$\frac{1}{C_1s}$
140	$C_1s(C_3s+G_2)$	$\frac{s^2C^2K_0^2de+s(1+A)K_0G_2Cf+G_1G_2}{s^2C^2K_0^2de+sC[(1+A)K_0G_2f-G_2]+G_1G_2}\ \frac{1}{C_1s}$	$\frac{1}{C_1s}$	$\frac{1}{C_1s}$
150	$C_1s(C_2s+C_3s+G_2)$	$\frac{s^2C^2K_0^2bd+s(1+A)K_0G_1Cd+G_1G_2}{s^2C^2K_0^2bd+sC[(1+A)K_0G_1d-G_2]+G_1G_2}\ \frac{1}{C_1s}$	$\frac{1}{C_1s}$	$\frac{1}{C_1s}$
152	$C_1s(C_2s+C_3s+G_3-\frac{1}{K_0}G_3)$	$\frac{s^2C^2K_0^2bd+s(1+A)K_0G_1Cd+G_1G_3}{s^2C^2K_0^2bd+sC[(1+A)K_0G_1d-G_3]+G_1G_3}\ \frac{1}{C_1s}$	$\frac{1}{C_1s}$	$\frac{1}{C_1s}$

Table 23. The Output Impedances of the Optimum Band-pass Sections

Circuit of Figure	$Z_{out}(s)$	$\lim_{s\to 0} Z_{out}(s)$	$\lim_{s\to\infty} Z_{out}(s)$
9	0	0	0
10	0	0	0
13	0	0	0
14	0	0	0
32	$\dfrac{Cs(1+dK_0)+G_1}{s^2C^2K_0^2b+s(1+A)K_0G_2Cd+G_1G_2}$	R_2	$\dfrac{1+dK_0}{sCK_0^2b}$
34	$\dfrac{C_1s+G(1+dK_0)}{s^2C_1C_2+s(1+A)K_0C_2Gd+G^2K_0^2b}$	$\dfrac{1+dK_0}{GK_0^2b}$	$\dfrac{1}{sC_2}$

Circuit of Figure	$Z_{out}(s)$	$\lim_{s\to 0} Z_{out}(s)$	$\lim_{s\to\infty} Z_{out}(s)$
64	$\dfrac{G_1+Cfs}{s^2C^2K_0^2f+s(1+A)K_0CG_1d+G_1G_2}$	R_2	$\dfrac{1}{sCK_0^2}$
68	$\dfrac{C_1s+Gf}{s^2C_1C_2+(1+A)K_0GC_1d+G^2K_0^2f}$	$\dfrac{1}{GfK_0^2}$	$\dfrac{1}{sC_2}$
72	$\dfrac{G_1+sC[b(1+K_0)+f]}{s^2C^2K_0^2f+s(1+A)K_0G_2Cb+G_1G_2}$	R_2	$\dfrac{bK_0+f}{sCK_0^2f}$
76	$\dfrac{C_1s+G[b(1+K_0)+f]}{s^2C_1C_2+s(1+A)K_0C_2Gb+G^2K_0^2f}$	$\dfrac{bK_0+f}{GK_0^2f}$	$\dfrac{1}{sC_2}$
79	0	0	0
81	0	0	0
84	0	0	0
86	0	0	0

Table 24. The Output Impedances of the Optimum Low-pass Sections

Circuit of Figure	$Z_{out}(s)$	$\lim_{s\to 0} Z_{out}(s)$	$\lim_{s\to\infty} Z_{out}(s)$
88	0	0	0
100	$\frac{sC_1+G(1+b+fK_0)}{s^2C_1C_2+s(1+A)K_0C_2Gf+G^2K_0^2be}$	$\frac{1+b+fK_0}{GK_0^2be}$	$\frac{1}{sC_2}$
128	0	0	0
136	0	0	0
144	$\frac{sC_1+G(1+b)}{s^2C_1C_2+s(1+A)K_0C_1Gd+G^2K_0^2bd}$	$\frac{1+b}{GK_0^2bd}$	$\frac{1}{sC_2}$
146	$\frac{sC_1+G(1+b)}{s^2C_1C_3+s(1+A)K_0C_1Gd+G^2K_0^2bd}$	$\frac{1+b}{GK_0^2bd}$	$\frac{1}{sC_3}$

Table 25. The Output Impedances of the Optimum High-pass Sections

Circuit of Figure	$Z_{out}(s)$	$\lim_{s\to 0} Z_{out}(s)$	$\lim_{s\to\infty} Z_{out}(s)$
90	0	0	0
102	$\frac{sC(1+b+fK_0)+G_1}{s^2C^2K_0^2be+s(1+A)K_0G_2Cf+G_1G_2}$	R_2	$\frac{1+b+fK_0}{sCK_0^2be}$
132	0	0	0
140	0	0	0
150	$\frac{sC(1+b)+G_1}{s^2C^2K_0^2bd+s(1+A)K_0G_1Cd+G_1G_2}$	R_2	$\frac{1+b}{sCK_0^2bd}$
152	$\frac{sC(1+b)+G_1}{s^2C^2K_0^2bd+s(1+A)K_0G_1Cd+G_1G_3}$	R_3	$\frac{1+b}{sCK_0^2bd}$

CHAPTER VII

EXAMPLES AND EXPERIMENTAL RESULTS

In this chapter, examples of the derived band-pass, low-pass, and high-pass sections will be presented. Actual circuits were built in the laboratory and tested to show that the realization procedures which have been developed are not only correct but also practical.

Example 1: Use the circuit of Figures 9 to realize an active-tunable band-pass section with $Q_0 = 5$ and the changes in Q_0 when the center-frequency is tuned from 100 Hz to 250 Hz not to exceed five percent.

Solution:

From Table 17,

$$N = \frac{f_1}{f_0} = \frac{250}{100} = 2.5$$

$$\frac{\Delta Q_1}{Q_0} = \frac{(1-N)A}{1+NA}\ , \ A > 0$$

$$-0.05 = \frac{(1-2.5)A}{1+a.5A}\ , \ A = 0.03636$$

$$K_0 = 2Q_0\frac{1+A}{A} = 2\times5\times\frac{1+0.03636}{0.03636} = 285.0$$

$$K_N = \frac{K_0}{N} = \frac{285.0}{2.5} = 114.0$$

We let $R_1 = 1000$ ohms, $b = 100$

$$R_3 = R_1\left(Q_0(1+A)-1\right) = 1000(5\times1.03636-1) = 4181.8 \text{ ohms}$$

$$R_2 = \frac{R_3 b}{Q_0(1+A)} = \frac{4181.8\times100}{5\times1.03636} = 80702 \text{ ohms}$$

$$C_1 = \frac{1}{\omega_0 R_2 K_0} = \frac{1}{2\times\pi\times100\times80702\times285} = 69.198\times10^{-12} \text{ farads}$$

$$C_2 = bC_1 = 10^2\times69.198\times10^{-12} = 6919.8\times10^{-12} \text{ farads}$$

$$|A_0| = \frac{Q_0(1+A)-1}{1+A} = \frac{5\times1.03636-1}{1.03636} = 4.034$$

The normalized frequency responses of the gain when the output is taken from terminal 4 are shown in Figure 154. When the controlled gains are tuned from 285.0 to 114.0, there is no noticeable change in the phase angle; however, a small change is found in the magniudes in the neighborhood of the center-frequencies. As can be seen from Figure 154, the percentage bandwidth is almost constant when the center-frequency is tuned from 100 Hz to 250 Hz, and thus results in very slightly change in Q_0.

The input impedances of the circuit are found by Table 20 and are shown in Figure 155. When the controlled gains are tuned from 285.0 to

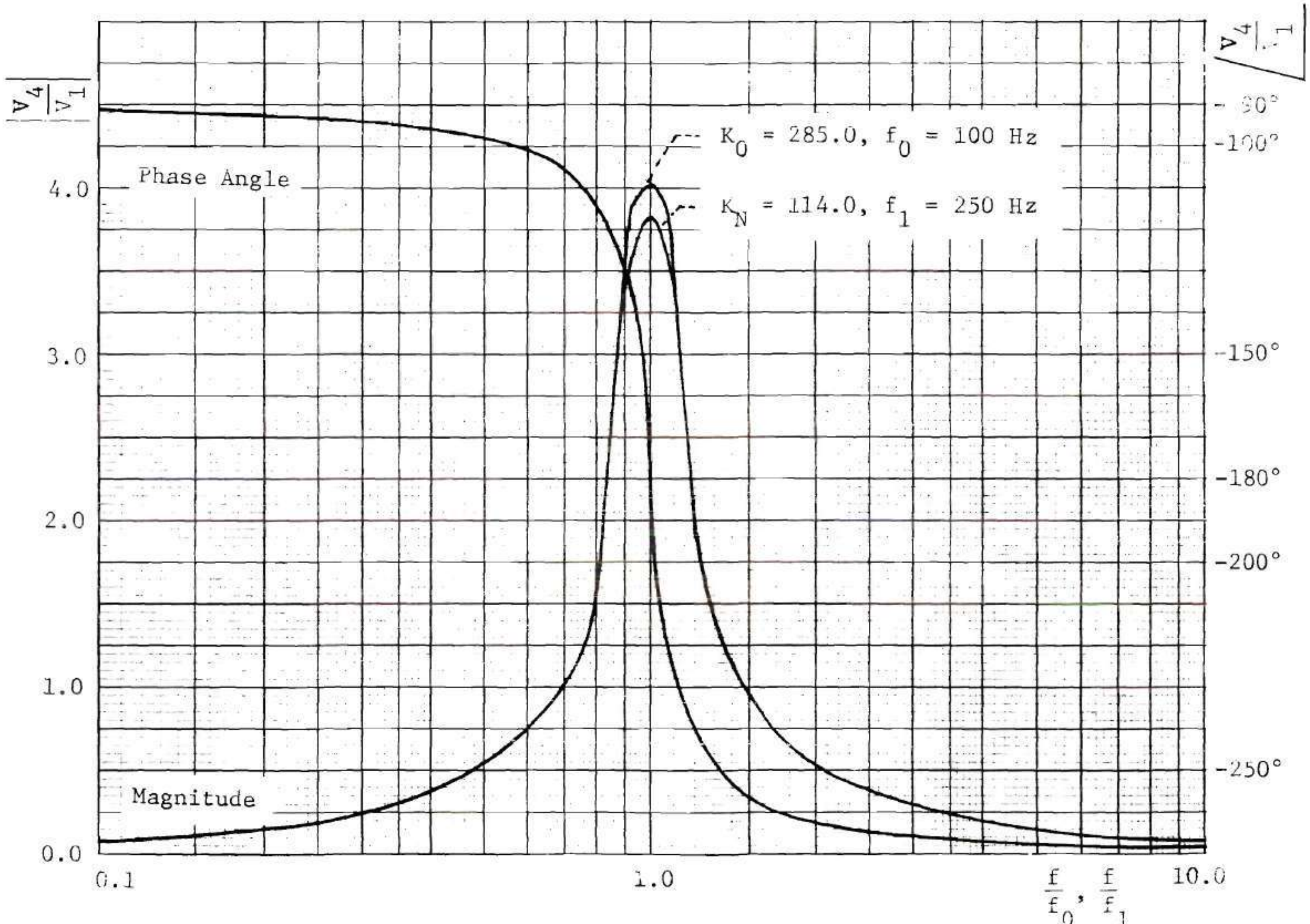


Figure 154. The Frequency Response of Example 1

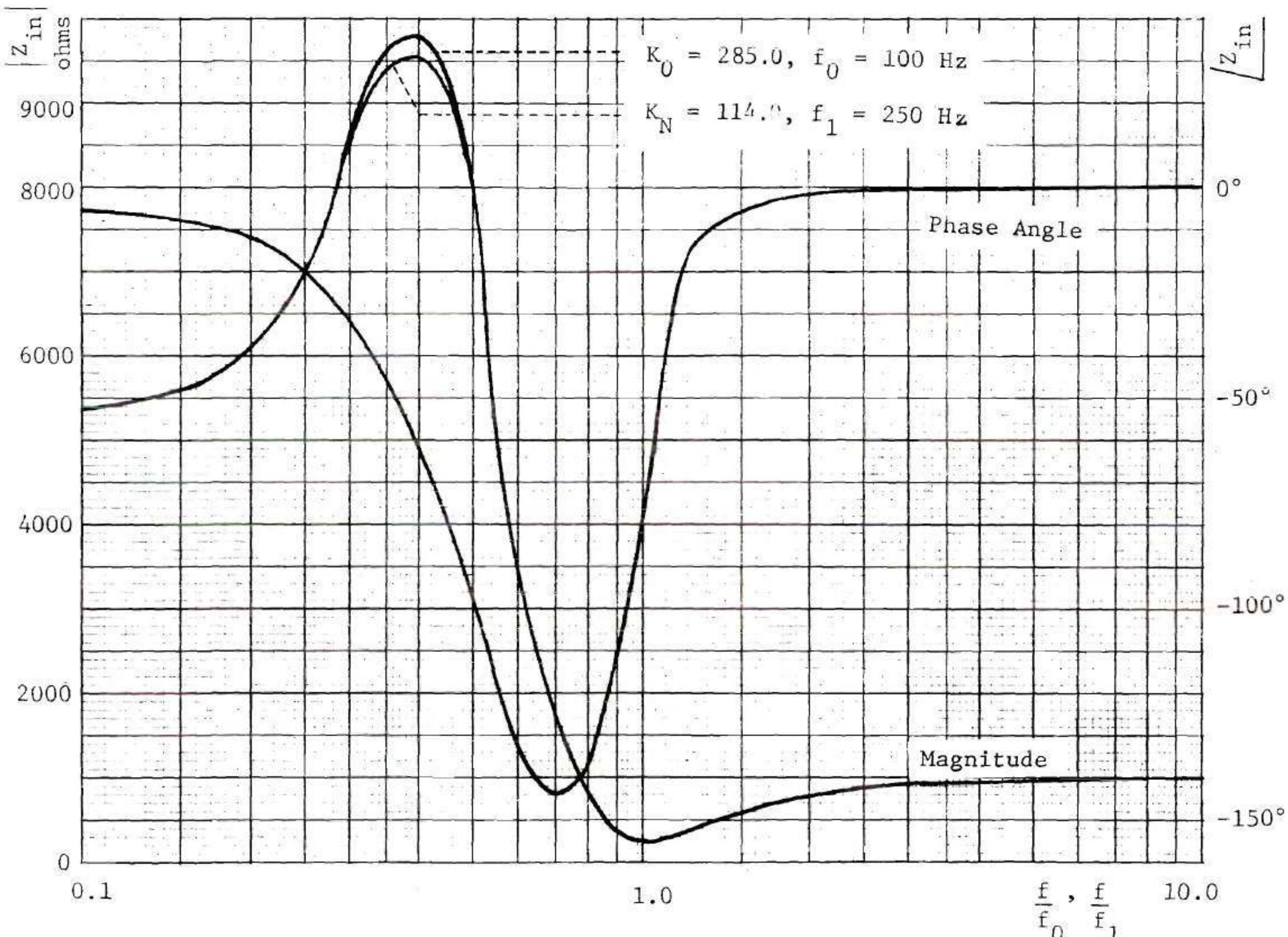


Figure 155. The Input Impedance of Example 1

114.0, changes in both the phase angle and the magnitude of the input impedance are very small. From Figure 155, only changes in the magnitudes are noticeable. The result can be expected from the equation for $Z_{in}(s)$ in Table 20 since we use controlled gains that are very much larger than unity.

The output is taken from the output of a VCVS, therefore this circuit has a zero output impedance.

Example 2: Use the circuit of Figure 88 to realize an active-tunable maximally-flat-response* low-pass section with the changes in Q_0 when the cut-off frequency is tuned from 1000 Hz to 2500 Hz not to exceed five percent.

Solution:

From Table 18,

$$N = \frac{f_1}{f_0} = \frac{2500}{1000} = 2.5$$

$$\frac{\Delta Q_1}{Q_0} = \frac{A(N-1)}{N+A}, \quad A > 0$$

$$0.05 = \frac{A(2.5-1)}{2.5+A}, \quad A = 0.0862$$

$$M = Q_0(1+A) = 0.707 \times 1.0862 = 0.768$$

*Equivalent to $Q = \frac{1}{\sqrt{2}}$

We let $R_1 = 100$ ohms, $d = 0.1$, $b = 0.001$

$$K_0 = \frac{1+M^2(1+1/d)}{M} Q_0 \frac{1+A}{A} = \frac{1+0.59\times 11}{0.768} \times .707 \times \frac{1.0862}{0.0862} = 86.874$$

$$K_N = NK_0 = 2.5\times 86.874 = 217.19$$

$$C_1 = \frac{dK_0}{\omega_0 MR_1} = \frac{0.1\times 86.874}{2\times\pi\times 10^3\times 0.768\times 10^2} = 18.0017\times 10^{-6} \text{ farads}$$

$$C_2 = \frac{M^2C_1b}{d} = \frac{0.59\times 18.0017\times 10^{-6}\times 10^{-3}}{10^{-1}} = 0.106196\times 10^{-6} \text{ farads}$$

$$R_2 = \frac{R_1}{b} = 100\times 10^3 = 100\text{K ohms}$$

$$R_3 = \frac{R_1}{d} = 100\times 10 = 1\text{K ohms}$$

$$|A_0(\omega_0)| = 0.707\ |A_0(\omega=0)| \simeq 0.707\times \frac{R_3}{R_1} = 7.07$$

The normalized frequency-responses of the gain when the output is taken from terminal 5 are shown in Figure 156. When the controlled gains are tuned from 86.874 to 217.19, there is no noticeable change in the phase angle; however, a small change is found in the magnitudes at the neighborhood of the cut-off frequencies. The percentage change in the magnitudes of the gains at the cut-off frequencies is 5 percent which is equal to the percentage changes in Q_0.

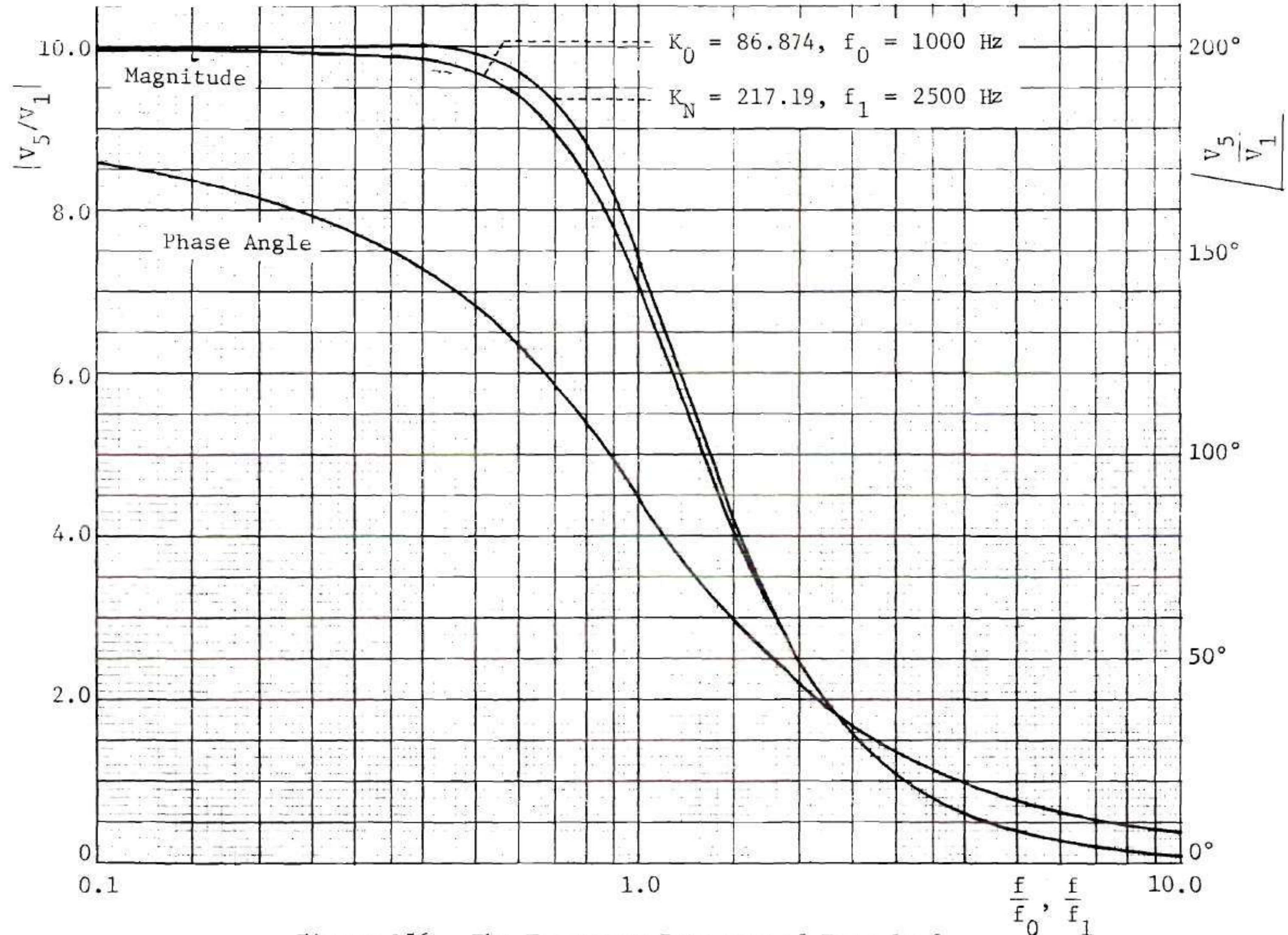


Figure 156. The Frequency Response of Example 2

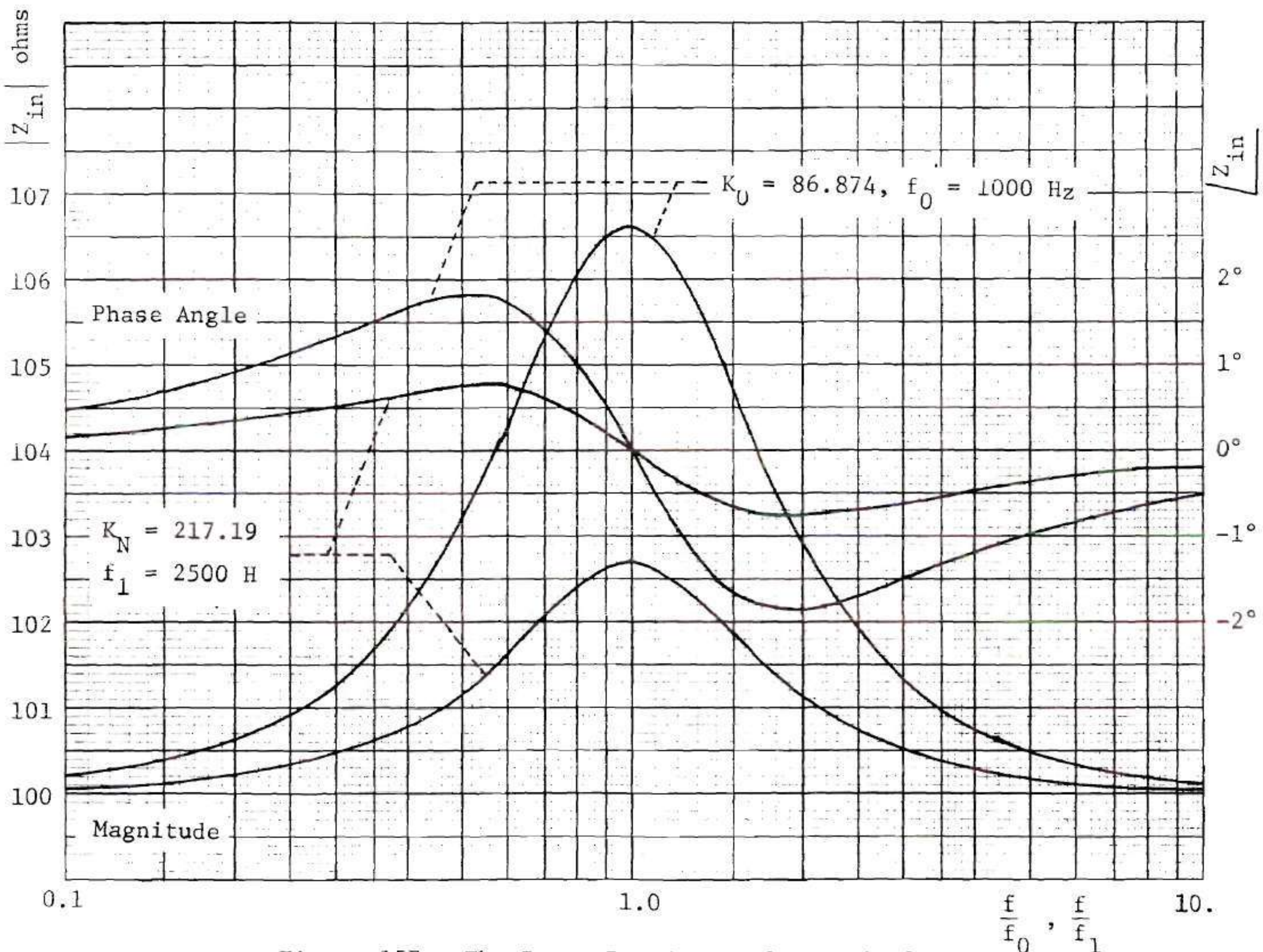


Figure 157. The Input Impedance of Example 2

The input impedances of the circuit are found by using Table 21 and are shown in Figure 157. The input impedances have very small phase angles and, thus, can be assumed to be purely resistive. The magnitude variation stays within 7 percent when the controlled gain is 86.874, and within 3 percent when the controlled gain is 217.19.

The output is taken from the output of a VCVS. Therefore, this circuit has a zero output impedance.

Example 3: Use the circuit of Figure 90 to realize an active-tunable maximally-flat-response high-pass section with Q_0=5 and the changes in Q_0 when the cut-off frequency is tuned from 100 Hz to 250 Hz not to exceed 5 percent.

Solution:

From Table 19,

$$N = \frac{f_1}{f_0} = \frac{250}{100} = 2.5$$

$$\frac{\Delta Q_1}{Q_0} = \frac{(1-N)A}{1+N}, \; A > 0$$

$$-0.05 = \frac{(1-2.5)A}{1+2.5A}, \; A = 0.03636$$

$$M = Q_0(1+A) = 0.707\times 1.03636 = 0.733$$

We let R_1 = 100 ohms, d = 0.1, b = 0.001

$$K_0 = \frac{1+M^2(1+1/d)}{M} Q_0 \frac{1+A}{A} = \frac{1+0.538\times 11}{0.733}\times 0.707\times\frac{1.03636}{0.03636} = 189.95$$

$$K_N = \frac{K_0}{N} = \frac{189.95}{2.5} = 75.98$$

$$C_1 = \frac{M}{dR_1\omega_0K_0} = \frac{0.733}{0.1\times10^2\times2\times\pi\times10^2\times189.95} = 0.614014\times10^{-6} \text{ farads}$$

$$C_2 = bC_1 = 10^{-3}\times0.614014\times10^{-6} = 614.014\times10^{-12} \text{ farads}$$

$$C_3 = dC_1 = 10^{-1}\times0.614014\times10^{-6} = 0.0614\times10^{-6} \text{ farads}$$

$$R_2 = \frac{dR_1}{M^2} = \frac{0.1\times100}{0.538\times001} = 18621 \text{ ohms}$$

$$|A_0(\omega_0)| = 0.707\ |A_0(\omega_0 = \infty)| \approx 0.707\times\frac{C_1}{C_3} = 7.07$$

The normalized frequency responses of the gain when the output is taken from terminal 5 are shown in Figure 158. When the controlled gains are tuned from 189.95 to 75.98, there is no noticeable change in the phase angle; however, a small change is found in the magnitudes at the neighborhood of the cut-off frequencies. The percentage change in the magnitudes of the gains at the cut-off frequency is 5 percent which is equal to the percentage changes in Q_0.

The input impedances of the circuit are found by using Table 22 and shown in Figure 159. The input impedance has constant -90° phase angle, that is, capacitive impedance. As frequency increases, the magnitude of the input impedance decreases.

The output is taken from the output of a VCVS. Therefore this circuit has a zero output impedance.

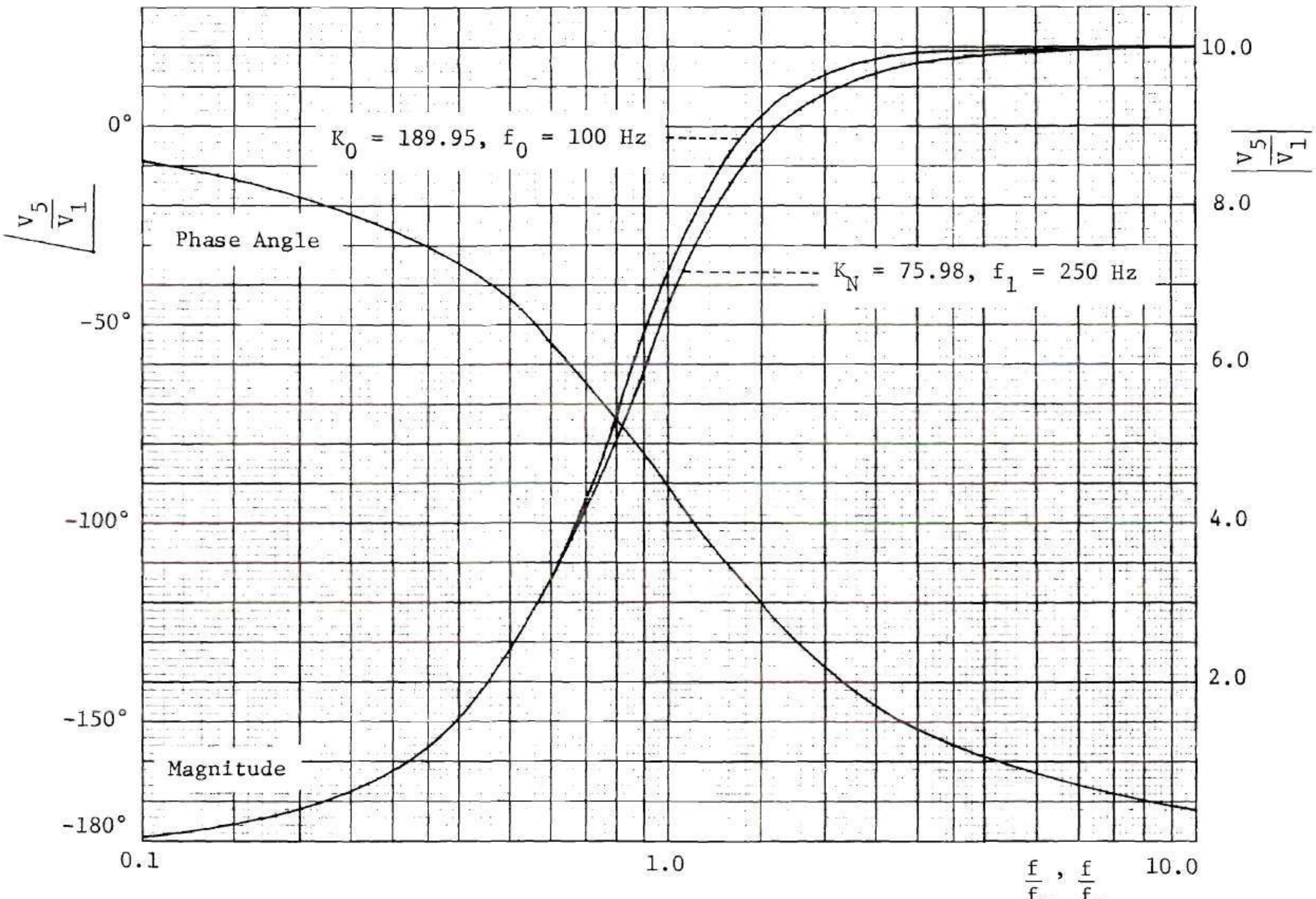


Figure 158. The Frequency Response of Example 3

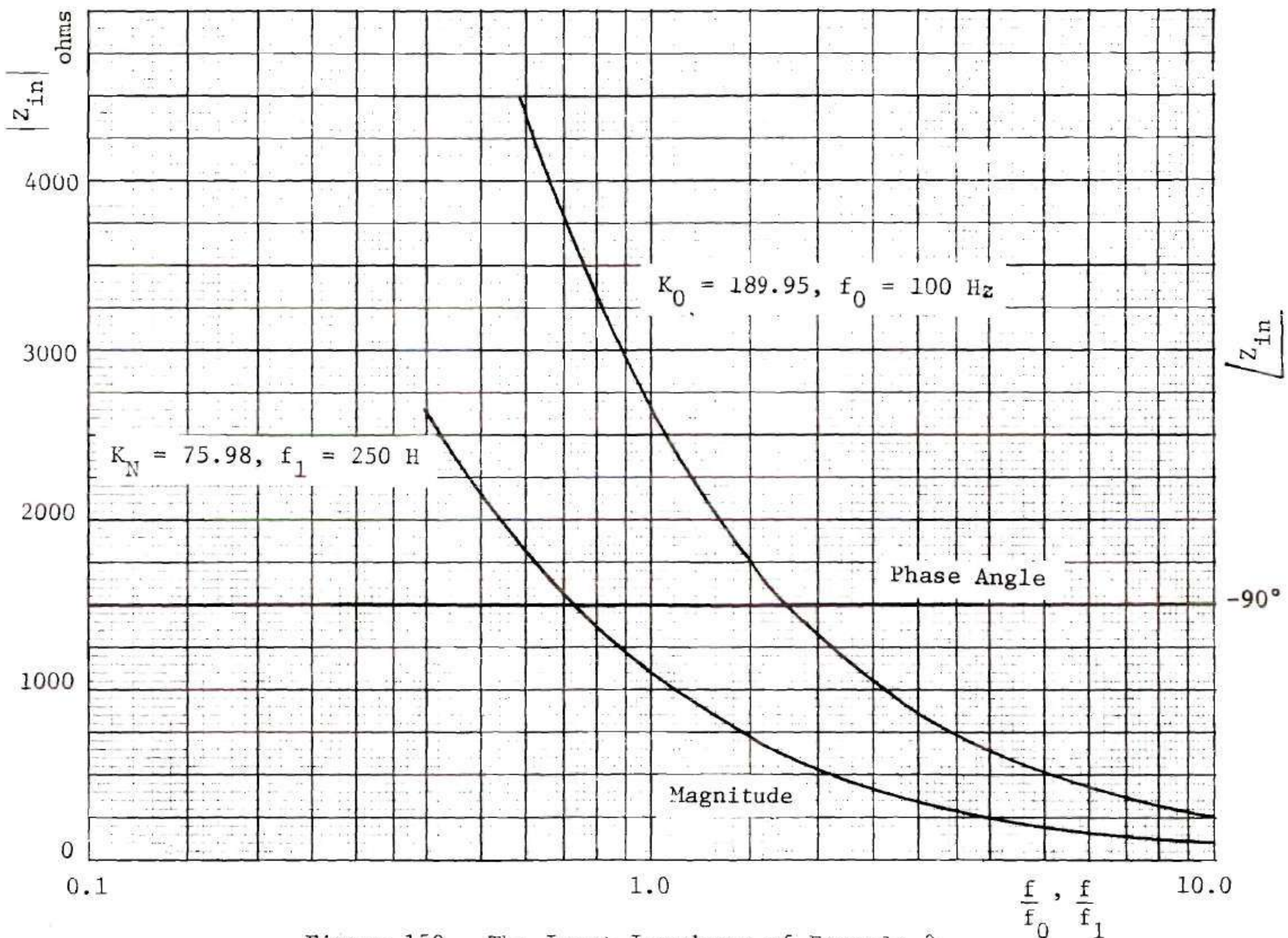


Figure 159. The Input Impedance of Example 3

Example 4: Construct and test an actual circuit of Figure 9 to realize an active-tunable band-pass section with $Q_0 = 5$ and the changes in Q_0 when the center-frequency is tuned from 101 Hz to 202 Hz not to exceed 20 percent.

Commercial operational amplifiers are chosen for the VCVS. Two Motorola MC1741CG internally compensated operational amplifiers are used together with the following passive elements.

$$\begin{aligned} R_1 &= 100 \quad \text{ohms} \\ R_2 &= 8450 \quad \text{ohms} \\ R_3 &= 560 \quad \text{ohms} \\ C_1 &= 0.00464\times10^{-6} \text{ farads} \\ C_2 &= 0.464\times10^{-6} \text{ farads} \end{aligned}$$

The controlled gains are mechanically tuned by changing the feedback resistances in the universal circuits for realizing the inverting and non-inverting amplifiers. The actual circuit is shown in Figure 160.

Data for the circuit of Figure 160 are taken in the laboratory and compared to the theoretical results. Outputs are taken from terminal 5 so that the high-gain outputs can be easily measured.

A comparison of the theoretical and the experimental results is shown in Figure 161. The theoretical values of the center-frequencies when the controlled gains are 20, 25, 30, 35, and 40 are 202.55 Hz, 162.04 Hz, 135.03 Hz, 115.74 Hz, and 101.27 Hz. The experimental results are very close to the theoretical ones.

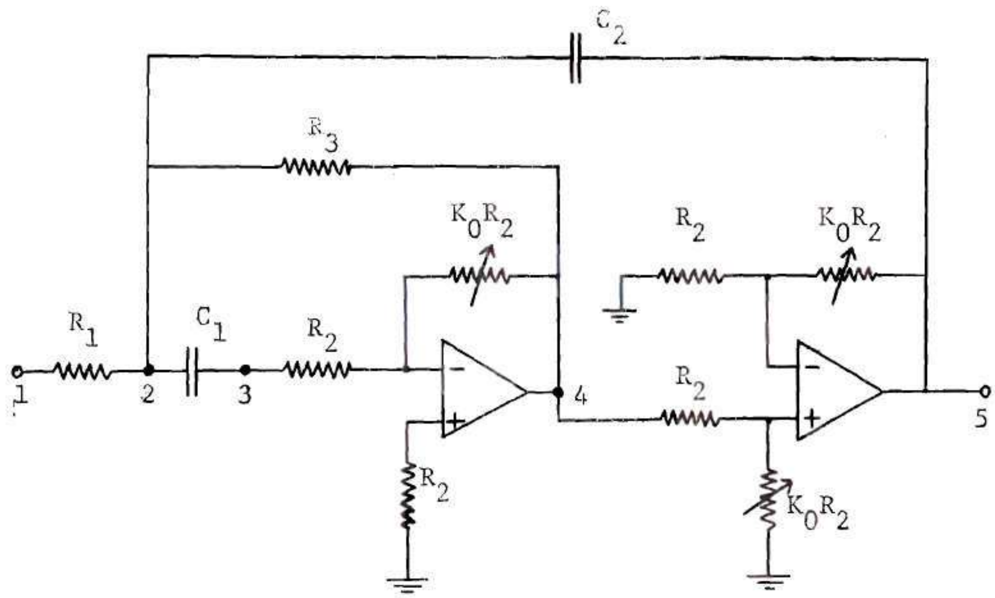


Figure 160. The Test Circuit

Some noises are present in the circuit. This is because the voltage at termian 3 is very low. However, since the purpose of this experiment is to show that the realization of the section is feasible and we have obtained good experimental results, no attempt in improving the VCVS is made.

The relationships between A, $\frac{1+A}{A}$, $\frac{(1-N)A}{1+NA}$, and $\frac{A(N-1)}{N+A}$ are shown in Figures 162 and 163 so that we can easily find the minimum required gains of each circuit. If the center-frequency or the cut-off frequency of the section is proportional to the controlled gain, as for all derived high-pass sections and some of the derived band-pass sections, Figure 162 will be used. Figure 163 is used when the center-frequency or the cut-off frequency of the section is proportional to the inverse of the controlled gain, that is for all derived low-pass sections and some of the

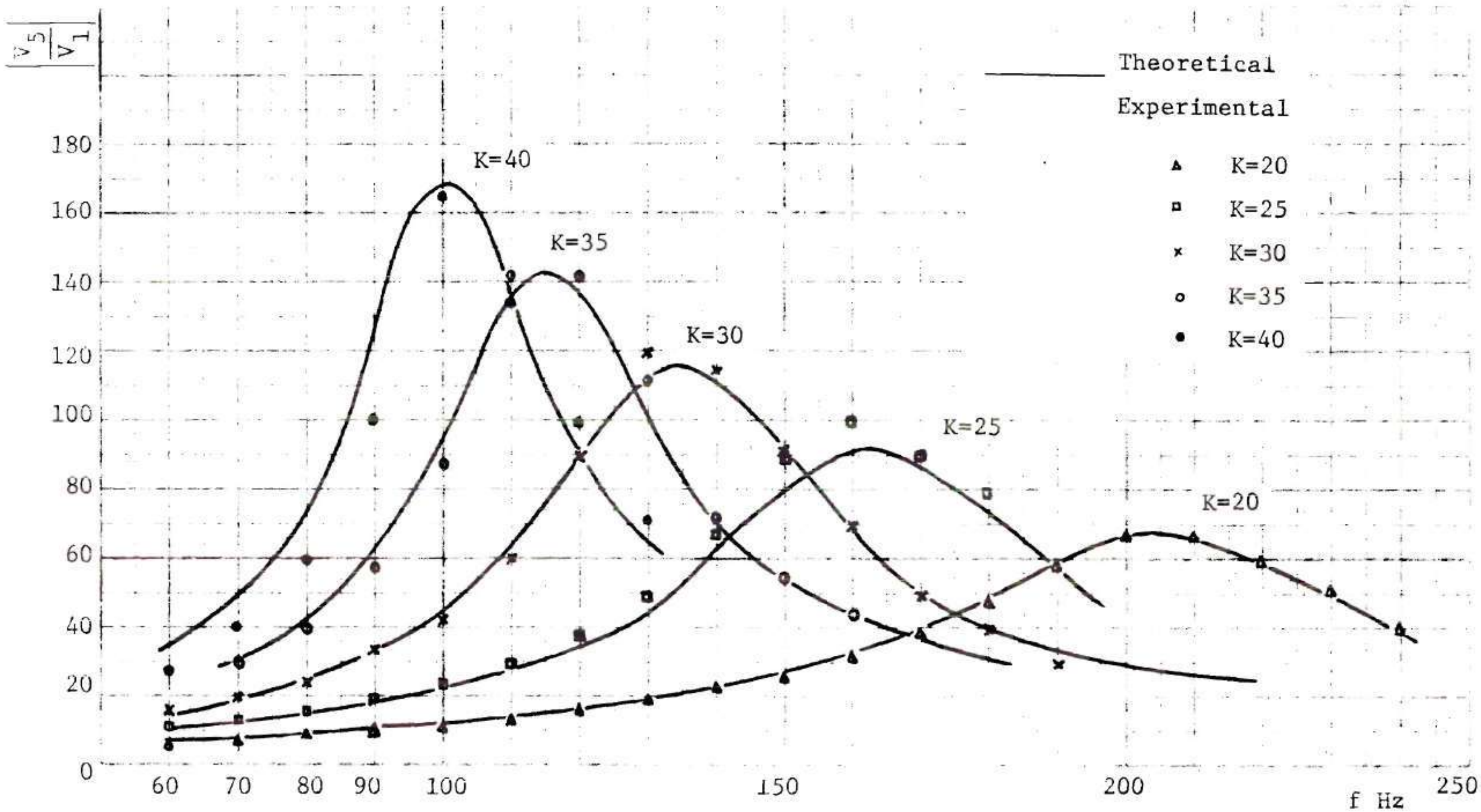


Figure 161. The Comparison of the Theoretical and the Experimental Results

derived band-pass sections. For example, we will use the circuit of Figure 9 with

$$Q_0 = 5$$

$$N = \frac{f_1}{f_0} = 2.0$$

$$\left|\frac{\Delta Q_1}{Q_0}\right| = 0.2$$

From Table 17, the center-frequency is proportional to the controlled gain and $\Delta Q_1/Q_0 = (1-N)A/(1+NA)$. We have to use Figure 162.

Corresponding to

$$\frac{(1-N)A}{1+NA} = 0.2 \text{ for } N = 2,$$

we need $A = 0.33$. The value of $(1+A)/A$ is also found in Figure 161 to be 4. Thus,

$$K_{0_{min}} = 2Q_0 \frac{1+A}{A} = 2\times5\times4 = 40$$

For other values of N not shown in Figures 162 and 163 and for better accuracy, the exact formula to find A and $(1+A)/A$ should be used.

When N increases, the minimum required gains do not increase linearly. For the same example, if $N = 4$, then $A = 0.091$ and $(1+A)/A = 12$. The minimum required gains increase to three times of that when $N = 2$.

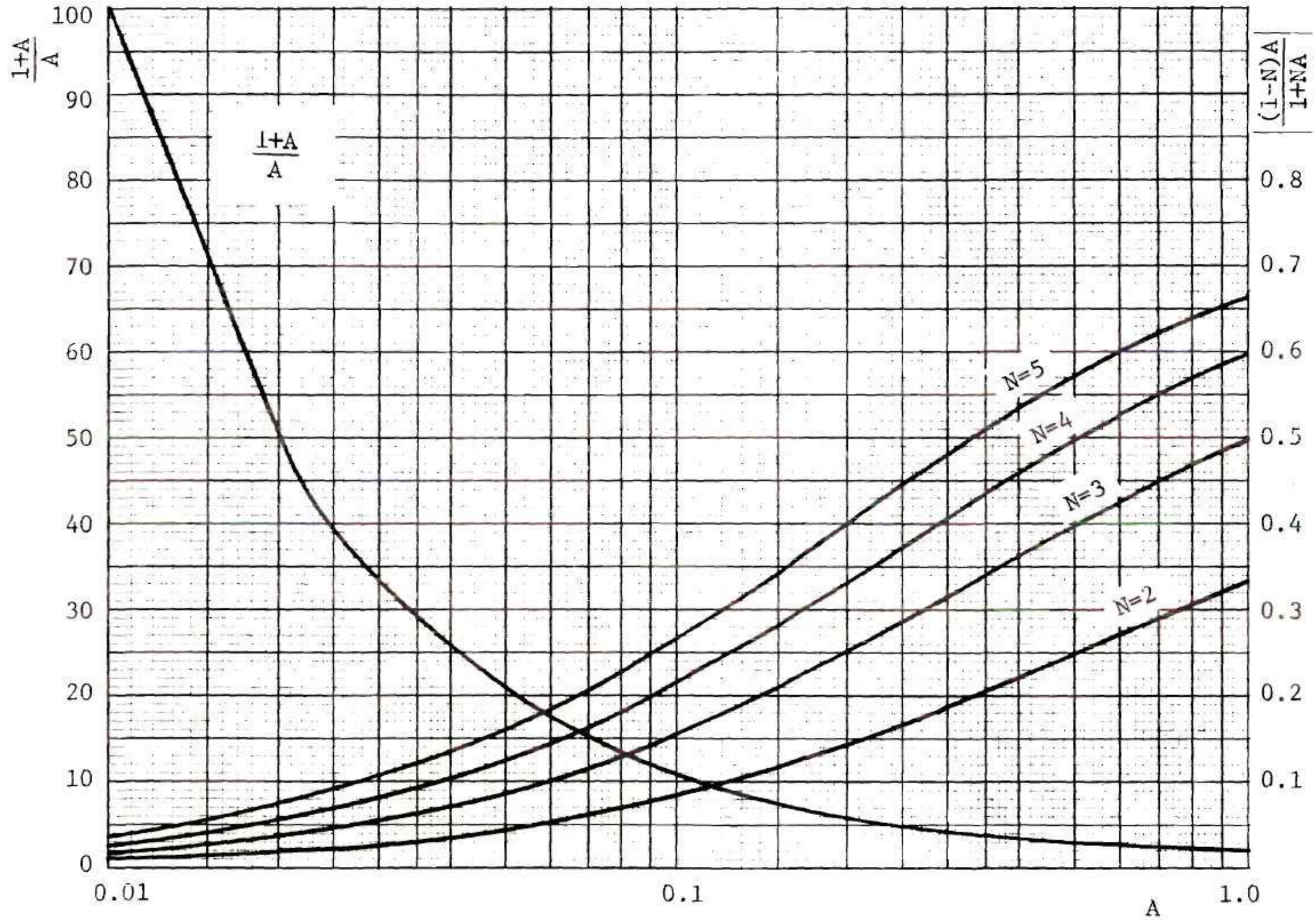


Figure 162. The Relationships of A, $\frac{1+A}{A}$, and $\frac{(1-N)A}{1+NA}$

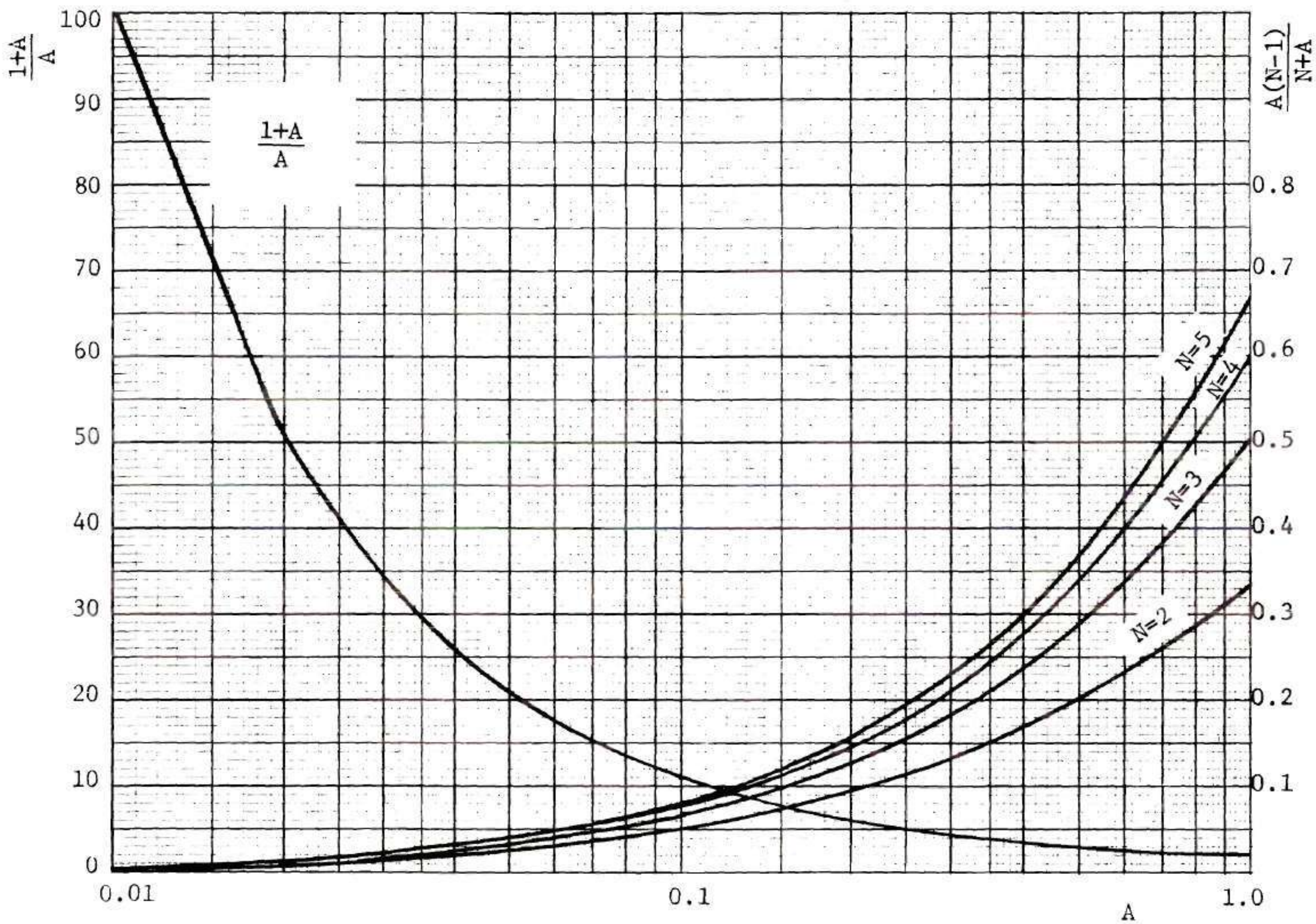


Figure 163. The Relationships of A, $\frac{1+A}{A}$, and $\frac{A(N-1)}{N+A}$

CHAPTER VIII

CONCLUSIONS AND RECOMMENDATIONS

It has been the purpose of this investigation to study a class of tunable active filters. The active filters of interest are realized by using VCVS as active elements. The center-frequency or the cut-off frequency of the filter is to be tuned within a range by varying the controlled gains of the VCVS.

The synthesis of the network is confined to that of second-order transfer functions as needed for low-pass, high-pass, or band-pass functions. This is to alleviate the high sensitivity of the pole positions to changes in the elements and arrive at simple circuits.

In the research, the VCVS are assumed to have ideal characteristics, that is, infinite input impedance, zero output impedance, zero reverse transmission, and ideal phase shift (either 0° or 180°). Passive elements are restricted to resistors and capacitors.

The first class of tunable filter, the constant bandwidth, can be easily realized by using only one VCVS. It is possible to tune the center-frequency or the cut-off frequency of the filter by varying the gain of the VCVS. One such circuit is shown in Figure 1. Thus, the realization of active tunable constant bandwidth filter is quite a simple one.

The other class of tunable filter, the constant percentage bandwidth, has been found particularly useful in the analyses of sound extending over a broad frequency range.[13,14,15] It is this class of filter that has been investigated in this research.

As a result of this investigation it was found that:

1. At least two VCVS and a five-node network are needed.

2. There is no practical advantage in increasing either the number of VCVS or the number of nodes.

3. The controlled gains, K_0's, of the VCVS should be kept at their minimum values so that the VCVS can be easily realized.

4. The ideal characteristics of the filters are approached when the controlled gains of the VCVS are very much larger than unity. However, finite values of K_0's have to be used in actual circuits and result in slight changes in Q_0 when the center-frequency or the cut-off frequency is tuned.

5. The minimum required gains can be obtained so that the change in Q_0 will be within a given tolerance.

6. The minimum required gains can be achieved with two VCVS and a five-node network.

7. The minimum required gains do not change when the number of nodes and the number of VCVS are increased.

8. Two VCVS and five-node network are used in realizing the filters.

The basic approach used in this research is described in Chapter II. The circuits applicable for band-pass section are derived in Chapter III. Chapter IV shows the circuits applicable for the low-pass and the high-pass sections. Eighty circuits are found for the band-pass sections, and thirty three circuits each are found for the low-pass and the high-pass sections.

The minimization on the controlled gains and on the number of passive elements are made in Chapter V. A group of optimum circuits are chosen and shown in Tables 17, 18, and 19. The input and output impedances of these circuits are derived in Chapter VI and are summarized in Tables 20, 21, and 22.

Several design examples are presented in Chapter VII. An actual circuit was built in the lbaoratory using two commercially available operational amplifiers. The experimental results are very close to the theoretical ones. Some difficulties were found in the experiment due to the saturation of the operational amplifiers when their peak-to-peak output voltages exceeded approximately 25 volts. The input voltage to the network had to be restricted. Noise also was appreciable in the network; however, the experiment was satisfactory. Improvement can be made by improving the VCVS. A noise and harmonic rejection circuit[16] could be used.

Further study can be made for other types of controlled sources, namely, the current controlled current source (CCCS), the current controlled voltage source (CCVS), and the voltage controlled current source (VCCS).

As shown in the examples, there are quantities that can be chosen, such as the values of the resistances and the ratio of the capacitances, in order to yield a circuit with the desired input impedance of the desired gain in the pass-band. When a certain criterion is given, these values can be optimally chosen. Computer-aided programs can also be used to simplify the design of the filters.

APPENDIX A

THE DERIVATION OF D(s) FOR THREE VCVS NETWORKS

For the proposed procedure of realizing the constant-Q tunable filters, we need the coefficient of s in the denominator of the voltage-transfer function to be in ascendent or descendent order of K_0. That is, if we have D(s) in quadratic form of s,

$$D(s) = P(K_0)(a_0+a_1K_0s+a_2K_0^2s^2) \qquad \text{(A-1)}$$

or $$D(s) = Q(K_0)(a_0'K_0^2+a_1'K_0s+a_2's^2) \qquad \text{(A-2)}$$

where $P(K_0)$ and $Q(K_0)$ are functions of K_0.

If only two VCVS are given, then in order to achieve D(s) as in (A-1) or (A-2), it is necessary that $P(K_0)$ and $Q(K_0)$ are constant. For example,

$$D(s) = a_0+a_1K_0s+a_2K_0^2s^2 \qquad \text{(A-3)}$$

or $$D(s) = a_0'K_0^2+a_1'K_0s+a_2's^2 \qquad \text{(A-4)}$$

Equations (A-3) and (A-4) show D(s) in the same form as the denominators of equations (5) and (9).

When three VCVS are given, D(s) of second-order or higher-order can be used. If we use D(s) of second-order, then $P(K_0)$ and $Q(K_0)$ will be constant and D(s) will have the same form as in (A-3) or (A-4). However, if we assume further that K_0 is very much larger than unity, we can let the coefficient of s^0 be the function of K_0^0 and K_0. That is

$$D(s) = (c_0+c_1K_0)+c_2K_0^2s+c_3K_0^3s^2 \tag{A-5}$$

We then approximate the coefficient of s^0 by the term containing K_0.

$$D(s) \simeq c_1K_0+c_2K_0^2s+c_3K_0^3s^2 \tag{A-6}$$

$$= K_0(c_1+c_2K_0s+c_3K_0^2s^2)$$

Equation (A-6) has the same form as (A-1) with $P(K_0)=K_0$. We can also let the coefficient of s^2 be the function of K_0^0 and K_0.

$$D(s) = c_0'K_0^3+c_1'K_0^2s+(c_2'K_0+c_3')s^2 \tag{A-7}$$

We then approximate the coefficient of s^2 by the term containing K_0.

$$D(s) \simeq c_0'K_0^3+c_1'K_0^2s+c_2'K_0s^2 \tag{A-8}$$

$$= K_0(c_0'K_0^2+c_1'K_0s+c_2's^2)$$

Equation (A-8) has the same form as (A-2) with $Q(K_0) = K_0$.

If D(s) is a third-order function of s, then with three VCVS, we can choose D(s) to be

$$D(s) = b_0+b_1K_0s+b_2K_0^2s^2+b_3K_0^3s^3 \tag{A-9}$$

or

$$D(s) = b_0'K_0^3+b_1'K_0^2s+b_2'K_0s^2+b_3's^3 \tag{A-10}$$

Since we are interested only in the second-order D(s) by the reasoning as in Chapter II, D(s) of third- or higher-order will be disregarded.

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